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# Nucleon Parton Distributions at Low Normalization Point in the Large $N_c$ Limit

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**Abstract**

At large $N_c$ the nucleon can be viewed as a soliton of the effective chiral lagrangian. This picture of nucleons allows a consistent nonperturbative calculation of the leading-twist parton distributions at a low normalization point. We derive general formulae for the polarized and unpolarized distributions (singlet and non-singlet) in the chiral quark-soliton model. The consistency of our approach is demonstrated by checking the baryon number, isospin and total momentum sum rules, as well as the Bjorken sum rule. We present numerical estimates of the quark and antiquark distributions and find reasonable agreement with parametrizations of the data at a low normalization point. In particular, we obtain a sizeable fraction of antiquarks, in agreement with the phenomenological analysis.

# Contents

# 1 Introduction

The distributions of quarks, antiquarks and gluons in nucleons, as measured in the inclusive deep inelastic scattering of leptons, provides us probably with the largest portion of quantitative information about strong interactions. Until now only the *evolution* of the structure functions from a high value of $q^2$ to even higher values, has been successfully compared with the data. It is the field of the perturbative QCD, and its success has been, historically, essential in establishing the validity of the QCD itself. Unfortunately, the initial conditions for that evolution, namely the leading-twist distributions at a relatively low normalization point, belong to the field of the nonperturbative QCD, and the success here is still rather modest.

In this paper we attempt to calculate parton distributions at a low normalization point in the limit of large number of colours, $N_c \to \infty$. Even though in reality $N_c = 3$, the academic limit of large $N_c$ is known to be a useful guideline. At large $N_c$ the nucleon is heavy and can be viewed as a classical soliton of the pion field [1]. An example of the dynamical realization of this idea is given by the Skyrme model [2]. However, the Skyrme model is based on an unrealistic effective chiral lagrangian. A far more realistic effective chiral lagrangian is given by the functional integral over quarks in the background pion field [3, 4]:

$$\exp\left(iS_{\text{eff}}[\pi(x)]\right) = \int D\psi D\bar{\psi} \exp\left(i\int d^4x\bar{\psi}(i\partial\!\!\!/ - MU^{\gamma_5})\psi\right),$$

$$U = \exp\left(i\pi^a(x)\tau^a\right), \qquad U^{\gamma_5} = \exp\left(i\pi^a(x)\tau^a\gamma_5\right) = \frac{1+\gamma_5}{2}U + \frac{1-\gamma_5}{2}U^\dagger. \tag{1.1}$$

Here $\psi$ is the quark field, $M$ is the effective quark mass which is due to the spontaneous breakdown of chiral symmetry (generally speaking, it is momentum-dependent) and $U$ is the $SU(2)$ chiral pion field. The effective chiral action given by eq. (1.1) is known to contain automatically the Wess–Zumino term and the four-derivative Gasser–Leutwyler terms, with correct coefficients. Therefore, at least the first four terms of the gradient expansion of the effective chiral lagrangian are correctly reproduced by eq. (1.1), and chiral symmetry arguments do not leave much freedom to further modifications. Eq. (1.1) has been derived from the instanton model of the QCD vacuum [4, 5], which provides a natural mechanism of chiral symmetry breaking and enables one to express the dynamical mass $M$ and the ultraviolet cutoff $\Lambda$ intrinsic in eq. (1.1) through the $\Lambda_{QCD}$ parameter. The effective chiral theory (1.1) is valid for the values of the quark momenta up to the ultraviolet cutoff $\Lambda$. Therefore, in using eq. (1.1) we imply that we are computing the parton distributions at the normalization point about $\Lambda \approx 600$ MeV. It should be mentioned that eq. (1.1) is of a general nature: one need not believe in instantons and still use eq. (1.1).

An immediate implication of this effective chiral theory is the quark-soliton model for baryons of ref. [6], which is in the spirit of the earlier works [7, 8] but without the vacuum instability paradox noticed there. According to the model nucleons can be viewed as $N_c$ (=3) "valence" quarks bound by a self-consistent hedgehog-like pion field (the "soliton") whose energy coincides in fact with the aggregate energy of quarks from the negative-energy Dirac continuum. Similarly to the Skyrme model large $N_c$ are needed as an algebraic parameter to justify the use of the mean-field approximation (like one needs large $Z$ to justify the Thomas–Fermi atom), however the $1/N_c$ corrections can be and in some cases are computed [9, 10]. The quark-soliton model of

nucleons developed in ref. [6] includes a collective-quantization procedure to deal with the rotational excitations of the quark-pion soliton. (The quantization of the otherwise static solution is necessary to obtain physical baryon states with definite quantum numbers). It enables one to calculate the $N$ and $\Delta$ properties, such as formfactors, $\Delta - N$ splitting, magnetic moments, axial constants, etc. For a review of baryon properties obtained from the model see [11] and references therein. Until now the model lacks explicit confinement (though probably it can be implemented along the lines discussed in ref. [12]), but it seems to be not so important for the *ground* state nucleon.

Turning to the calculation of the nucleon structure functions we note that the model possesses all features needed for a successful description of the nucleon parton structure: it is an essentially quantum field-theoretical relativistic model with explicit quark degrees of freedom, which allows an unambiguous identification of quark as well as antiquark distributions in the nucleon. This should be contrasted to the Skyrme model where it is not too clear how to define quark and antiquark distributions. The advantage of the model can be also seen if one compares it to any variant of the bag model. Unfortunately, the bag surface is not described consistently in terms of fields. Ignoring the "structure function of the surface" the bag models run into a violation of general theorems requiring the complete account for all constituents (see section 8). It should be added that all modern fits to the data tend to include antiquarks and gluons at a low normalization point, below 1 $GeV^2$ [13, 14, 15].

In this paper we develop the framework for calculating polarized and unpolarized quark and antiquark distributions of the nucleon as described by the effective chiral theory (1.1). We check the validity of general theorems, like the sum rules for the baryon number, isospin and the total momentum carried by partons, as well as the Bjorken sum rule for the polarized distributions. We also derive the expression for the Gottfried sum rule and show that its r.h.s. is generally non-zero.

We show that, from the viewpoint of large $N_c$, all distributions can be divided into "large" and "small" ones. We estimate numerically the "large" distributions: the singlet unpolarized distribution and the isovector polarized one, for quarks and antiquarks separately [1]. The obtained distributions should be, in principle, used as initial conditions for the standard perturbative evolution to higher values of $q^2$ where one can compare them with the available data. Actually, in this paper we compare our results with the parametrization of the data at a low normalization point performed recently by Glück, Reya *et al.* [15, 17].

[1] A few preliminary numerical results have been announced in ref. [12].

# 2 From QCD to the effective chiral theory

## 2.1 Light-cone representation for distribution functions

The unpolarized quark distribution function of flavour $f$ inside a nucleon with 4-momentum $P$, averaged over its spin, is given by the following QCD equation (see e.g. [18]):

$$q_f(x,\mu) = \frac{1}{4\pi}\int\limits_{-\infty}^{\infty} dz^- e^{ixp^+z^-}$$
$$\cdot\langle P|\bar\psi_f(0)\gamma^+ \mathrm{P}\exp\left\{-ig\int_0^z dz'^\alpha A_\alpha(z')\right\}\psi_f(z)|P\rangle\Big|_{z^+=0,\,z_\perp=0,\,\mu}, \tag{2.1}$$

where $\psi_f$ are quark fields and $A_\alpha$ is the gluon field. The antiquark distribution is

$$\bar q_f(x,\mu) = -\frac{1}{4\pi}\int\limits_{-\infty}^{\infty} dz^- e^{-ixp^+z^-}$$
$$\cdot\langle P|\bar\psi_f(0)\gamma^+ \mathrm{P}\exp\left\{-ig\int_0^z dz'^\alpha A_\alpha(z')\right\}\psi_f(z)|P\rangle\Big|_{z^+=0,\,z_\perp=0,\,\mu}. \tag{2.2}$$

Here we use the light-cone coordinates

$$z^\pm = \frac{z^0 \pm z^3}{\sqrt{2}}, \qquad \gamma^\pm = \frac{\gamma^0 \pm \gamma^3}{\sqrt{2}}, \tag{2.3}$$

and the nucleon state is normalized by

$$\langle P|P'\rangle = 2P^0\delta^3(\mathbf{P}-\mathbf{P}') \tag{2.4}$$

where $P$ is the nucleon 4-momentum. Throughout the paper $x$ is the Bjorken variable, $x = -q^2/(2P\cdot q)$, where $q$ is the 4-momentum transfer to the nucleon.

The physical meaning of these equations is clear: the parton model description of the nucleon is justified in the infinite momentum frame; by using a backward Lorentz transformation one can recover a nucleon at rest but the price is that one has to work with the quark correlation functions on the light cone. The transition from the infinite momentum frame to the light-cone correlation function over the nucleon at rest is helpful since in the large $N_c$ limit the nucleon is a heavy particle, and it is convenient to work in the nucleon rest frame.

The right hand sides of eqs. (2.1, 2.2) depend on the QCD normalization scale parameter $\mu$. Actually, the non-local light-cone products of fields in these equations should be considered as a formal compact notation for an expansion in a series of local operators, each of these operators being renormalized at the scale $\mu$. In contrast to QCD, the effective chiral field theory is nonrenormalizable and contains an explicit ultraviolet cutoff $\Lambda$ (not to be mixed up with the $\Lambda_{QCD}$ parameter). In the instanton vacuum model this cutoff appears as the inverse average instanton size: $\Lambda \approx \bar\rho^{-1} \approx$ 600 MeV [5]. The results obtained below refer thus to the low QCD scale $\mu$ of the order of 600 MeV.

We stress that we are computing the leading-twist distributions at a low normalization point and not the structure functions, observable in principle at low $q^2$. The former differ from the latter by higher-twist power corrections which are large at low $q^2$. The distributions we are computing can be directly used as initial conditions to the standard perturbative evolution to higher $q^2$ where the power corrections are suppressed so that the distributions become directly related to the observables.

Eqs.(2.1,2.2) allow one to introduce a single distribution function $q_f(x)$ defined in the interval $[-1,1]$ identifying it at negative $x$ with the antiquark distribution:

$$q_f(x) = \begin{cases} q_f(x), & x > 0\,, \\ -\bar{q}_f(-x), & x < 0\,. \end{cases} \tag{2.5}$$

With this definition of $q_f(x)$ eq. (2.1) is valid in the interval $-1 \le x \le 1$.

An alternative way to the structure functions is via their moments. For example, one defines the moments of the *singlet* structure function as

$$M_n = \int\limits_{-1}^{1} dx\, x^{n-1} \sum_f q_f(x)\,. \tag{2.6}$$

Introducing local quark and gluon twist-2 operators [19],

$$\mathcal{O}_n^{q,\mu_1\ldots\mu_n} = \frac{1}{2}\,\frac{i^{n-1}}{n!}\,[\bar\psi\gamma^{\mu_1}D^{\mu_2}\ldots D^{\mu_n}\psi + \text{ permutations} - \text{ traces }]\,, \tag{2.7}$$

$$\mathcal{O}_n^{G,\mu_1\ldots\mu_n} = \frac{i^{n-2}}{n!}\,\mathrm{Tr}\,[G^{\mu_1\nu}D^{\mu_2}\ldots D^{\mu_{n-1}}G_\nu{}^{\mu_n} + \text{ permutations} - \text{ traces }]\,, \tag{2.8}$$

one can express the moments of the singlet structure function as nucleon matrix elements of local operators:

$$M_n = \frac{i^{n-1}}{2} M_N^{-n}\langle P|\bar\psi_f \not{v}(v^\mu D_\mu)^{n-1}\psi_f|P\rangle, \tag{2.9}$$

where $v_\mu$ is a light-like vector,

$$v_\mu v^\mu = 0, \qquad v_\mu P^\mu = M_N. \tag{2.10}$$

When one starts to work with the effective low-energy theory (1.1) all information about gluons is already lost. Therefore, in order to recover the gluon distribution one has to go one step back, before the derivation of eq. (1.1) [4] and to rewrite the gluon operators in terms of (possibly nonlocal) quark operators. Ref. [16] explains how it can be done, and certain gluon operators are expressed there through quark fields only. Applying the method of ref. [16] to the twist-2 gluon operators (2.8) one observes, however, that in the leading order in the instanton packing fraction,

$$f = \bar\rho^4 \frac{N}{V} \ll 1, \tag{2.11}$$

these operators are zero. The reason is the $O(4)$ symmetry of the instanton field: after integration over instanton orientations one can build the $\mu_1\ldots\mu_n$ tensor only out of the Kronecker symbols, but it is impossible to get it traceless. In order to obtain a non-zero result one has to go beyond the zero-mode approximation of ref. [16] and/or

consider contributions of many instantons. Both ways would lead to extra powers of the packing fraction of instantons. Meanwhile, it is the smallness of this packing fraction which is used in the derivation of eq. (1.1). Eq. (1.1) contains actually two dimensional parameters: the constituent quark mass $M$ and the ultraviolet cutoff $\Lambda$; algebraically

$$\frac{M^2}{\Lambda^2} \sim f \ll 1. \tag{2.12}$$

Hence, to be consistent with the effective chiral lagrangian (1.1), one has to neglect the gluon operators, and to replace the covariant derivatives in eqs. (2.7, 2.9) by ordinary ones. That is what we are going to do in the rest of the paper. It should be kept in mind, however, that one *has* to introduce a finite cutoff $\Lambda$ in order to make the nucleon mass and some of the structure functions finite (fortunately, the potential divergences are but logarithmical). In fact gluons are resident in the "formfactor" of constituent quarks whose size is $\sim 1/\Lambda$. At $\Lambda \to \infty$ the constituent quarks are point-like, and there are no gluons. For the actual finite value of $\Lambda \approx 600$ MeV gluons have to show up. Unfortunately, the precise form of the gluon distribution depends, in the language of the chiral model, on the details of the ultraviolet regularization, therefore we shall not attempt to determine the gluon distribution in this paper.

In the "quarks-antiquarks only" approximation one is left with the following expression for the quark (and antiquark) distribution function $q_f(x)$ at $-1 < x < 1$:

$$q_f(x) = \frac{1}{4\pi}\int\limits_{-\infty}^{\infty} dz^- e^{ixp^+z^-} \langle P|\bar\psi_f(0)\gamma^+\psi_f(z)|P\rangle\Big|_{z^+=0,\, z_\perp=0}. \tag{2.13}$$

According to (2.3) one can write

$$\bar\psi_f(0)\gamma^+\psi_f(z) = \psi_f^\dagger(0)\gamma^0\gamma^+\psi_f(z) = \frac{1}{\sqrt{2}}\psi_f^\dagger(0)(1+\gamma^0\gamma^3)\psi_f(z)\,. \tag{2.14}$$

It follows from the constraint $z^+ = 0$ in (2.13) that $z^3 = -z^0$. Taking the nucleon at rest we have $P^+ = M_N/\sqrt{2}$, where $M_N$ is the nucleon mass. Therefore, the quark (and antiquark) distribution function (2.13) is

$$q_f(x) = \frac{1}{4\pi}\int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \langle P|\psi_f^+(0)(1+\gamma^0\gamma^3)\psi_f(z)|P\rangle\Big|_{z^3=-z^0,\, z_\perp=0}. \tag{2.15}$$

For the *polarized* distributions, $\Delta q_f(x)$, one has to insert the $\gamma_5$ matrix in this equation (see, e.g. [20]). In this paper we limit ourselves to two flavours, $u$ and $d$, neglecting altogether the strange quarks, therefore for both polarized and unpolarized distributions two cases have to be distinguished: flavour singlet and flavour nonsinglet or isovector. For isovector distributions one has to insert the $\tau^3$ matrix in eq. (2.15). All four distributions are considered in this paper.

Before turning to the calculation of these functions in the chiral quark-soliton model we make a general remark about the quark distributions in the limit $N_c \to \infty$.

In this limit the quark part of the nucleon momentum in the infinite momentum frame is distributed among $O(N_c)$ quarks and antiquarks so that each quark and antiquark carries $O(1/N_c)$ fraction of the nucleon momentum. This means that the quark

distribution function is concentrated at $x \sim 1/N_c$. Keeping in mind that the total number of quarks minus the number of antiquarks in the nucleon is $N_c$ we can write the baryon number sum rule:

$$\sum_f \int_{-1}^{1} dx q_f(x) = N_c \,. \tag{2.16}$$

Comparing this sum rule with the fact that quark distributions are concentrated at $x \sim 1/N_c$ we conclude that the singlet quark distribution has the following form in the large $N_c$ limit:

$$\sum_f q_f(x) = N_c^2 \rho(N_c x) \tag{2.17}$$

where the function $\rho(y)$ is stable in the limit $N_c \to \infty$. Similarly, the isovector polarized distribution, $\Delta u(x) - \Delta d(x)$, is normalized, via the Bjorken sum rule, to $g_A$ which, theoretically, is of the order of $N_c$. Therefore, this distribution has also the form of eq. (2.17). It should be contrasted to the isovector unpolarized distribution, $u(x)-d(x)$, normalized to the isospin which is $O(N_c^0)$. Therefore, it is $N_c$ times smaller than in eq. (2.17), and so is the singlet polarized distribution. In all four cases the antiquarks follow the same pattern as they are given by the same function as quarks but at $x < 0$. Exactly this behaviour of the distribution functions is obtained below.

## 2.2 Chiral quark-soliton model of the nucleon

Integrating out quarks in (1.1) one finds the effective chiral action,

$$S_{\rm eff}[\pi^a(x)] = -N_c \mathrm{Sp} \log D(U) \,, \qquad D(U) = i\partial_0 - H(U), \tag{2.18}$$

where $H(U)$ is the one-particle Dirac hamiltonian,

$$H(U) = -i\gamma^0 \gamma^k \partial_k + M\gamma^0 U^{\gamma_5} \,, \tag{2.19}$$

and Sp... is a functional trace.

For a given time-independent pion field $U = \exp(i\pi^a(\mathbf{x})\tau^a)$ one can find the spectrum of the Dirac hamiltonian,

$$H\Phi_n = E_n \Phi_n. \tag{2.20}$$

It contains the upper and lower Dirac continua (distorted by the presence of the external pion field), and, in principle, may contain discrete bound-state level(s), if the pion field is strong enough. If the pion field has the unity winding number, there is exactly one bound-state level which travels all the way from the upper to the lower Dirac continuum as one increases the spatial size of the pion field from zero to infinity [6]. One has to fill in this level to get a non-zero baryon number state. Since the pion field is colour blind, one can put $N_c$ quarks on that level in the antisymmetric state in colour. We denote the energy of the discrete level as $E_{\rm lev}, \;\; -M \le E_{\rm lev} \le M$.

The limit of large $N_c$ allows to use the mean-field approximation to find the nucleon mass – similarly to the Thomas–Fermi model of large $Z$ atoms. To get the nucleon mass one has to add up $N_c E_{\rm lev}$ and the energy of the pion field. Since the effective chiral

lagrangian is given by the determinant (2.18) the energy of the pion field coincides exactly with the aggregate energy of the lower Dirac continuum, the free continuum subtracted. The self-consistent pion field is thus found from the minimization of the functional [6]

$$M_N = \min_U N_c \left\{ E_{\rm lev}[U] + \sum_{E_n<0} (E_n[U] - E_n^{(0)}) \right\}. \tag{2.21}$$

From symmetry considerations one looks for the minimum in a hedgehog ansatz:

$$U_c(\mathbf{x}) = \exp\left[i\pi^a(\mathbf{x})\tau^a\right] = \exp\left[i(\mathbf{n}\cdot\tau)P(r)\right], \quad r = |\mathbf{x}|, \quad \mathbf{n} = \frac{\mathbf{x}}{r} \tag{2.22}$$

where $P(r)$ is called the profile of the soliton.

The minimum (2.21) is degenerate in respect to the translation of the soliton in space and to the rotation of the soliton field in ordinary and isospin spaces. For the hedgehog field (2.22) the two rotations are equivalent. Including slow rotations of the saddle-point pion field and quantizing it gives rise to the quantum numbers of the nucleon: its spin and isospin components [2, 6]. In order to take into account the translational and rotational zero modes one has to make a unitary rotation of the quark eigenfunctions and to shift their centre,

$$\Phi_n(\mathbf{x}) \to R\Phi_n(\mathbf{x}-\mathbf{X}), \tag{2.23}$$

and to make a projection to a concrete nucleon state under consideration. The projection into a nucleon state with given momentum $\mathbf{P}$ is obtained by integrating over all shifts $\mathbf{X}$ of the soliton,

$$\langle \mathbf{P}' | \ldots | \, \mathbf{P} \rangle = \int d^3\mathbf{X}\, e^{i(\mathbf{P}'-\mathbf{P})\cdot\mathbf{X}} \ldots \tag{2.24}$$

The projection to a nucleon with given spin ($S_3$) and isospin ($T_3$) components is obtained by integrating over all spin-isospin rotations $R$ of the soliton,

$$\langle S=T, S_3, T_3 | \ldots | S=T, S_3, T_3 \rangle = \int dR\, \phi^{\dagger\, S=T}_{S_3T_3}(R) \ldots \phi^{S=T}_{S_3T_3}(R), \tag{2.25}$$

where $\phi^{S=T}_{S_3T_3}(R)$ is the rotational wave function of the nucleon given by the Wigner finite-rotation matrix [6]:

$$\phi^{S=T}_{S_3T_3}(R) = \sqrt{2S+1}(-1)^{T+T_3} D^{S=T}_{-T_3,S_3}(R). \tag{2.26}$$

The four nucleon states have $S = T = 1/2$, with $S_3, T_3 = \pm 1/2$. [Taking the next rotational excitation with $S = T = 3/2$ one can as well compute the $\Delta$-resonance structure functions]. It is implied that $dR$ in eq. (2.25) is the Haar measure normalized to unity, $\int dR = \int d(AR) = \int d(RB) = 1$. In what follows we shall omit the superscript $S = T$.

## 2.3 Light-cone quark correlation functions in the nucleon

In this subsection we derive several equivalent ways of presenting the quark distribution function (2.15) in the chiral quark-soliton model. Depending on the circumstances one can choose a more convenient representation.

Eq. (2.15) is a matrix element of a non-local quark bilinear operator over the nucleon state with definite 4-momentum $P$ and spin and isospin components. According to the previous subsection one can write down a general equation for such matrix elements; the time dependence of the quark operators is accounted for by the energy exponents. We write explicitly all the flavour ($f, g = 1, 2$) and the Dirac ($i, j = 1, ..., 4$) indices for clarity:

$$\langle \mathbf{P}, S_3, T_3 | \psi^\dagger_{fi}(x^0, \mathbf{x}) \psi^{gj}(y^0, \mathbf{y}) | \mathbf{P}, S_3, T_3 \rangle = 2P_0 N_c \int d^3\mathbf{X} \int dR\, \phi^\dagger_{S_3T_3}(R) \cdot \sum_{\substack{n \\ \text{occup.}}} \exp[iE_n(x^0 - y^0)] \Phi^\dagger_{n,f'i}(\mathbf{x} - \mathbf{X}) R^{\dagger f'}_{f} R^g_{g'} \Phi^{g'j}_n(\mathbf{y} - \mathbf{X}) \phi_{S_3T_3}(R) . \quad (2.27)$$

The functions $\Phi_n$ are eigenstates of energy $E_n$ of the Dirac hamiltonian (2.19) in the external (self-consistent) pion field $U_c$. Summation over colour indices is implied in the quark bilinears, hence the factor $N_c$ in the r.h.s.

In eq. (2.27) the quark is first annihilated in the nucleon by the operator $\psi(\mathbf{y})$ and only then created by the operator $\psi^\dagger(\mathbf{x})$. Therefore the sum goes over occupied states. For the opposite ordering of the quark field operators, the order of creation and annihilation is opposite and the sum runs over non-occupied states:

$$\langle \mathbf{P}, S_3, T_3 | \psi^{gj}(y^0, \mathbf{y}) \psi^\dagger_{fi}(x^0, \mathbf{x}) | \mathbf{P}, S_3, T_3 \rangle = 2P_0 N_c \int d^3\mathbf{X} \int dR\, \phi^\dagger_{S_3T_3}(R) \cdot \sum_{\substack{n \\ \text{non-occup.}}} \exp[iE_n(x^0 - y^0)] \Phi^\dagger_{n,f'i}(\mathbf{x} - \mathbf{X}) R^{\dagger f'}_{f} R^g_{g'} \Phi^{g'j}_n(\mathbf{y} - \mathbf{X}) \phi_{S_3,T_3}(R) \quad (2.28)$$

We note that the eigenfunctions of the Dirac hamiltonian form a complete set of functions only when both occupied and non-occupied states are taken into account:

$$\sum_{\substack{n \\ \text{all}}} \Phi^\dagger_{n,fi}(\mathbf{x}) \Phi^{gj}_n(\mathbf{y}) = \delta^g_f \delta^j_i \delta(\mathbf{x} - \mathbf{y}). \quad (2.29)$$

Adding up eqs. (2.27, 2.28) at $x^0 = y^0$ and using the completeness condition (2.29) we observe that these equations are compatible with the standard equal-time anticommutator,

$$\left\{ \psi^\dagger_{fi}(\mathbf{x}), \psi^{gj}(\mathbf{y}) \right\} = \delta^g_f \delta^j_i \delta(\mathbf{x} - \mathbf{y}) . \quad (2.30)$$

In a Lorentz-invariant field theory the fermion operators should anticommute for any space-like separation:

$$\left\{ \psi(y), \psi^\dagger(x) \right\} = 0, \qquad \text{if } (x - y)^2 < 0 . \quad (2.31)$$

Our starting point, eq. (2.15), is a nucleon matrix element of a product of quark operators with a *light-like* separation which should be understood as a limit of *space-like*

separations. Indeed, the light-like separation is obtained from the infinite momentum frame. As long as the momentum is large but finite one has a space-like separation. Therefore, the $\psi$, $\psi^\dagger$ operators in eq. (2.15) anticommute, and two alternative though equivalent representations (2.27, 2.28) can be written for the distribution functions; one sums over the occupied states, the other sums over non-occupied states of the Dirac hamiltonian.

Finally, there exists another representation for the structure functions, this time through the imaginary part of the Feynman Green function in the background pion field. Let $G(\omega_1, \mathbf{p_1};\ \omega_2, \mathbf{p_2})$ be the Fourier transform of the Feynman two-point Green function

$$\begin{aligned} G(x^0, \mathbf{x};\ y^0, \mathbf{y}) &= -\langle y^0, \mathbf{y}| \frac{1}{i\partial\!\!\!/ - MU^{\gamma_5}} |x^0, \mathbf{x}\rangle \\ &= \langle y^0, \mathbf{y}|(i\partial\!\!\!/ + MU^{-\gamma_5}) \frac{1}{\partial^2 + M^2 - iM(\partial\!\!\!/ U^{-\gamma_5}) - i0} |x^0, \mathbf{x}\rangle, \end{aligned} \tag{2.32}$$

the free Green function being

$$G^{(0)}(p_1; p_2) = (2\pi)^4 \delta^{(4)}(p_1 - p_2) \frac{M + p\!\!\!/_1}{M^2 - p_1^2 - i0}. \tag{2.33}$$

In principle, one can expand the Green function (2.32) in powers of the pion field, $U - 1$, and its derivatives, $\partial U$. For the time-independent pion field $U$ one has

$$G(p_1^0, \mathbf{p_1};\ p_2^0, \mathbf{p_2}) = 2\pi\delta(p_1^0 - p_2^0) S(p_1^0, \mathbf{p_1}, \mathbf{p_2}). \tag{2.34}$$

Using this Green function we derive in Appendix A the following representation for the singlet distribution function:

$$\sum_f q_f(x) = -\ \mathrm{Im}\ \frac{N_c M_N}{2\pi} \int \frac{d^4 p}{(2\pi)^4}\ 2\pi\delta(p^0 + p^3 - xM_N)\ \mathrm{Tr}[S(p^0, \mathbf{p}, \mathbf{p})(\gamma^0 + \gamma^3)]. \tag{2.35}$$

Since the Green function $S(p^0, \mathbf{p}, \mathbf{p})$ can be expanded in the pion field and/or its derivatives one can use this representation to make a quick estimate of the structure function, see section 7.

# 3 Singlet unpolarized distribution

## 3.1 Sum over quark levels

We start with the flavour singlet distribution function, $u(x)+d(x)$, and the antiquark one, $\bar{u}(x)+\bar{d}(x)$. The singlet case is simpler than the isovector one since one can ignore the rotation of the soliton. Indeed in that case the indices of the orientation matrices in eq. (2.27) are contracted, $R^\dagger R = 1$, and the integral over orientations $R$ in eq. (2.27) becomes trivial and equal to unity. Taking the nucleon at rest, $\mathbf{P} = 0,\ P_0 = M_N$, and using eq. (2.27) for a specific light-cone separation of quarks operator as suggested by eq. (2.15) we get:

$$u(x)+d(x) = \frac{N_c M_N}{2\pi} \int d^3\mathbf{X} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \sum_{\substack{n \\ \text{occup.}}} e^{-iE_n z^0}$$
$$\cdot \Phi^\dagger_{n,fi}(-\mathbf{X})(1+\gamma^0\gamma^3)^i_j \Phi^{fj}_n(\mathbf{z}-\mathbf{X})\Big|_{z^3=-z^0,\ z_\perp=0}\,. \tag{3.1}$$

We remind the reader that the same equation gives, at $x<0$, the antiquark distribution, $\bar{u}(x)+\bar{d}(x) = -(u(-x)+d(-x))$. Passing to the momentum representation,

$$\Phi_n(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \Phi_n(\mathbf{p}) e^{i(\mathbf{p}\cdot\mathbf{x})}, \tag{3.2}$$

and integrating over the coordinates of the nucleon center of inertia, $\mathbf{X}$, we obtain:

$$u(x)+d(x) = \frac{N_c M_N}{2\pi} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \sum_{\substack{n \\ \text{occup.}}} e^{-iE_n z^0} \langle n|(1+\gamma^0\gamma^3)\exp(-iz^0 p^3)|n\rangle$$
$$= N_c M_N \sum_{\substack{n \\ \text{occup.}}} \langle n|(1+\gamma^0\gamma^3)\delta(E_n + p^3 - xM_N)|n\rangle\,, \quad x\in[-1,1], \tag{3.3}$$

where $|n\rangle = \Phi_n(\mathbf{p})$; taking the matrix element implies integration over $d^3\mathbf{p}/(2\pi)^3$. However, eq. (3.3) can be understood in any other basis provided $p^k = -i\partial/\partial x^k$ is the momentum operator.

An alternative representation for the distribution function arises from eq. (2.28) where summation goes over *non-occupied* states:

$$u(x)+d(x) = -\frac{N_c M_N}{2\pi} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \sum_{\substack{n \\ \text{non-occup.}}} e^{-iE_n z^0} \langle n|(1+\gamma^0\gamma^3)\exp(-iz^0 p^3)|n\rangle$$
$$= -N_c M_N \sum_{\substack{n \\ \text{non-occup.}}} \langle n|(1+\gamma^0\gamma^3)\delta(E_n + p^3 - xM_N)|n\rangle\,, \quad x\in[-1,1]. \tag{3.4}$$

Both representations, over occupied and non-occupied states of the Dirac hamiltonian, should be absolutely equivalent were the theory finite. However, the effective chiral action (2.18) implies an ultraviolet cutoff: the nucleon mass itself and its structure functions diverge logarithmically. In order to support the equivalence of two

representations, eq. (3.3) and eq. (3.4), the ultraviolet regularization should be introduced in such a way as to preserve the anticommutativity of the quark fields at a space-like separation. In particular, it means that the completeness of the fermion states, leading to the equal-time anticommutator (2.29) must not be violated. For example, a relativistically-invariant regularization by the Pauli–Villars method preserves the equivalence of eqs. (3.3, 3.4) and other necessary properties of the distribution functions. On the contrary, a naive energy cutoff in the Dirac continua violates the completeness, and is hence unacceptable.

In all above formulae we implied the vacuum subtraction: whenever we write $\langle N|A|N\rangle$ with a local operator $A$, we mean in fact $\langle N|A - \langle 0|A|0\rangle|N\rangle$. Without this subtraction $\langle N|A|N\rangle$ contains a divergence proportional to $\delta^{(3)}(0)$. The vacuum subtraction is effectively equivalent to the subtraction in eqs. (3.3, 3.4) of the appropriate sums over eigenstates $|n^{(0)}\rangle$ with eigenenergies $E_n^{(0)}$ of the free Dirac operator $H_0$.

For the free Dirac hamiltonian one has obviously $E_n^{(0)} = \pm\sqrt{|\mathbf{p}_n|^2 + M^2}$. Therefore the delta function $\delta(xM_N - E_n^{(0)} - p_n^3)$ vanishes if sign $(x) = -\mathrm{sign} E_n^{(0)}$. Hence in eq. (3.3) the vacuum subtraction term vanishes at $x > 0$ (that is for quarks) whereas in eq. (3.4) the vacuum subtraction is not needed at $x < 0$, that is for antiquarks.

Both quark and antiquark distribution functions must be positive. At $x > 0$ the function $u(x) + d(x)$ is given by eq. (3.3) without vacuum subtraction. We can rewrite this equation as

$$u(x) + d(x) = \frac{N_c M_N}{2\pi} \int \frac{d^2 p_\perp}{(2\pi)^2} \sum_{\substack{n \\ \text{occup.}}} \left[\Phi_n^\dagger(\mathbf{p})(1 + \gamma^0\gamma^3)\Phi_n(\mathbf{p})\right]_{p^3 = xM_N - E_n}, \quad x > 0. \tag{3.5}$$

This expression is explicitly positive because $(1 + \gamma^0\gamma^3)/2$ is an orthogonal projector.

Similarly, for the antiquark distribution one can write an explicitly positive expression:

$$\begin{aligned}\bar{u}(x) + \bar{d}(x) &= -[u(-x) + d(-x)] \\ &= \frac{N_c M_N}{2\pi} \int \frac{d^2 p_\perp}{(2\pi)^2} \sum_{\substack{n \\ \text{non-occup.}}} \left[\Phi_n^\dagger(\mathbf{p})(1 + \gamma^0\gamma^3)\Phi_n(\mathbf{p})\right]_{p^3 = -xM_N - E_n}, \quad x > 0\end{aligned} \tag{3.6}$$

Although these formulae are explicitly positive one should keep in mind that they are ultraviolet divergent. In principle, one can regularize them so that the positivity is preserved. However it is not obvious *a priori* that one can find a regularization that simultaneously preserves both positivity and the equivalence of eq. (3.3) with eq. (3.4). We have observed that the Pauli–Villars method is favoured in this respect, too.

## 3.2 Trace representation for distribution functions

All the above representations for quark distribution functions are written as sums of diagonal matrix elements of certain operators over either occupied or non-occupied eigenstates of the one-particle Dirac hamiltonian $H$. Therefore one can easily rewrite these sums as operator traces. To this end we apply to eq. (3.3) the spectral decomposition

$$\sum_{\substack{n\\ \text{occup.}}} |n\rangle\langle n| e^{-iz^0 E_n} = \int\limits_{-\infty}^{E_{\text{lev}}+0} d\omega\; \delta(\omega - H) e^{-iz^0\omega} \tag{3.7}$$

and obtain

$$u(x)+d(x) = N_c M_N \int\limits_{-\infty}^{E_{\text{lev}}+0} d\omega\; \mathrm{Sp}\left[\delta(\omega - H)\delta(\omega + p^3 - xM_N)(1+\gamma^0\gamma^3)\right] - (H \to H_0)\,, \tag{3.8}$$

where Sp... is the functional trace.

Similarly, starting from the quark distribution function in the form of the sum over non-occupied states, eq. (3.4), we arrive to the following representation:

$$u(x)+d(x) = -N_c M_N \int\limits_{E_{\text{lev}}+0}^{+\infty} d\omega\; \mathrm{Sp}\left[\delta(\omega - H)\delta(\omega + p^3 - xM_N)(1+\gamma^0\gamma^3)\right] - (H \to H_0)\,. \tag{3.9}$$

Both formulae are valid at $-1 < x < 1$. We remind the reader that, according to the results of the previous subsection, in eq. (3.8) one has to make a vacuum subtraction at $x < 0$ whereas in eq. (3.9) the vacuum subtraction is needed at $x > 0$.

Symbolical as they may seem, eqs. (3.8, 3.9) give a practical way of computing the structure functions. To saturate the functional trace one can use any complete set of functions. For example, the quark eigenfunctions of the Dirac hamiltonian (2.20) may be used. In this basis the hamiltonian $H$ is diagonal but the momentum $p^3$ is not. Another way is to use the eigenfunctions of the free hamiltonian, the so-called Kahana–Ripka basis [21]. The trace representation is also helpful in deriving general relations in a laconic form, see below.

## 3.3 Baryon number sum rule

Let us show that the baryon number sum rule is automatically satisfied in the above equations. To get the baryon number sum rule one has to integrate (3.8) over $x$ from $-1$ to 1. In the r.h.s. of that equation $x$ enters through the product $xM_N$ where $M_N = O(N_c)$, so that in the large $N_c$ limit we can replace this integral by the integral over the whole real axis of $x$, which leads to the following result:

$$\int\limits_{-1}^{1} dx\,[u(x) + d(x)] = N_c \int\limits_{-\infty}^{E_{\text{lev}}+0} d\omega\; \mathrm{Sp}\left[\delta(\omega - H)(1+\gamma^0\gamma^3)\right] - (H \to H_0)\,. \tag{3.10}$$

Owing to the rotational hedgehog symmetry of the soliton the term $\gamma^0\gamma^3$ gives no contribution, and we are left with

$$\int\limits_{-1}^{1} dx\,[u(x) + d(x)] = N_c\, \mathrm{Sp}\left[\theta(-H + E_{\text{lev}} + 0) - \theta(-H_0)\right], \tag{3.11}$$

which counts the number of the filled levels of the Dirac hamiltonian, the number of the levels in the free lower Dirac continuum subtracted. According to ref. [6] it is the baryon number of the state. We have thus proved the sum rule

$$\int_{-1}^{1} dx\,[u(x)+d(x)] = \int_{0}^{1} dx[u(x)+d(x)-\bar{u}(x)-\bar{d}(x)] = N_c B \tag{3.12}$$

where $B$ is the baryon number of the state; $B=1$ for the nucleon.

## 3.4 Moments of distribution functions

We define the moments of the singlet structure function as

$$M_n = \int_{-1}^{1} dx\, x^{n-1} \sum_f q_f(x)\,. \tag{3.13}$$

Let us multiply (3.8) by $x^{n-1}$ and integrate over $x$. In the large $N_c$ limit one can extend the integration region to $-\infty < x < \infty$. Integrating the quark distribution in the form of eq. (3.8) we obtain the following representation for the moments:

$$M_n = N_c M_N^{1-n} \int_{-\infty}^{E_{\rm lev}+0} d\omega\; \mathrm{Sp}\left[\delta(\omega - H)(\omega + p^3)^{n-1}(1+\gamma^0\gamma^3)\right] - (H \to H_0)\,. \tag{3.14}$$

Note that we need the vacuum subtraction here since this result is derived by integrating (3.8) over both positive and negative $x$'s.

Similarly, the representation (3.9) based on the summation over non-occupied states leads to the alternative expression for the moments:

$$M_n = -N_c M_N^{1-n} \int_{E_{\rm lev}+0}^{+\infty} d\omega\; \mathrm{Sp}\left[\delta(\omega - H)(\omega + p^3)^{n-1}(1+\gamma^0\gamma^3)\right] - (H \to H_0). \tag{3.15}$$

Both representations in fact follow from a third representation where the integration over $\omega$ goes along the imaginary axis, to the right of $E_{\rm lev}$. Putting $\omega = i\omega'$ we can write

$$M_n = -iN_c M_N^{1-n} \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \mathrm{Sp}\left[\frac{1}{\omega' + iH}(i\omega' + p^3)^{n-1}(1+\gamma^0\gamma^3)\right] - (H \to H_0)\,. \tag{3.16}$$

Indeed, closing the $\omega'$ integration contour to the upper cut (corresponding to the lower Dirac continuum plus the discrete level) or to the lower cut and pole (corresponding to the upper Dirac continuum) one immediately reproduces eqs. (3.14, 3.15). Note that we assume that the $\omega'$ integration contour in eq. (3.16) is chosen so that the discrete level belongs to the occupied states.

The fact that different deformations of the $\omega'$ integration contour in eq. (3.16) lead to the two representations in terms of summation over occupied and non-occupied

states respectively, gives another proof of the equivalence of these equations. We remind the reader that earlier we have derived this equivalence from causality, see subsection 2.3. It illustrates a general connection between causality and analyticity.

However, an arbitrary ultraviolet regularization of the theory may easily destroy both causality and analyticity. One has to check that a particular regularization does not induce new singularities in the $\omega'$ plane, and that it does not prevent one from closing the contours to the upper or to the lower cuts. For example, the popular proper time regularization would violate the last requirement. On the contrary, the Pauli-Villars regularization does not spoil the analytic properties, as well as casuality, see subsection 3.1.

## 3.5 Momentum sum rule

Let us derive the momentum sum rule for the quark distribution functions. It holds true only if equations of motion are satisfied, i.e. the pion field $U$ which binds quarks in the nucleon is the minimum of the functional (2.21) which can be symbolically written as

$$M_N = N_c \,\mathrm{Sp}\,[\theta(E_{\mathrm{lev}} - H + 0)H] - (H \to H_0). \tag{3.17}$$

The saddle-point equation for the self-consistent pion field reads:

$$\mathrm{Sp}[\theta(E_{\mathrm{lev}} + 0 - H)\delta_U H] = 0\,. \tag{3.18}$$

where $\delta_U H$ is an arbitrary variation of the Dirac hamiltonian under a variation of the chiral field $U$. Let us consider a particular (dilatational) variation of the chiral field, $U(x) \to U[(1+\xi)x]$ with an infinitesimal $\xi$ such that $\delta U = \xi x^k \partial_k U$. Then the correspondent variation of the Dirac Hamiltonian is

$$\delta_U H = M\gamma^0 \xi x^k \partial_k U^{\gamma_5} = \xi[x^k\partial_k, M\gamma^0 U^{\gamma_5}] = \xi\left([x^k\partial_k, H] - i\gamma^0\gamma^k\partial_k\right).$$

Inserting this variation into the saddle-point equation (3.18) and taking into account that

$$\mathrm{Sp}(\theta(E_{\mathrm{lev}} + 0 - H)[x^k\partial_k, H]) = \mathrm{Sp}([H, \theta(E_{\mathrm{lev}} + 0 - H)]x^k\partial_k) = 0,$$

we get a useful identity:

$$\mathrm{Sp}(\theta(E_{\mathrm{lev}} + 0 - H)\gamma^0\gamma_k\partial_k) = 0\,. \tag{3.19}$$

Owing to the hedgehog symmetry of the self-consistent pion field one can write a general tensor

$$\mathrm{Sp}(\theta(E_{\mathrm{lev}} + 0 - H)\gamma^0\gamma_k\partial_l) = \delta_{kl}A \tag{3.20}$$

with the zero constant $A$ because of eq. (3.19).

Having made the necessary preparations, we turn to the second moment of the structure function $M_2$. Using the representation (3.14) for the moments we can write

$$\begin{aligned}
M_2 &= N_c M_N^{-1} \int\limits_{-\infty}^{E_{\rm lev}+0} d\omega \;\mathrm{Sp}\left[\delta(\omega - H)(\omega + p^3)(1+\gamma^0\gamma^3)\right] - (H \to H_0) \\
&= N_c M_N^{-1} \mathrm{Sp}\left[\theta(E_{\rm lev} - H + 0)(H + p^3)(1+\gamma^0\gamma^3)\right] - (H \to H_0) \\
&= N_c M_N^{-1} \mathrm{Sp}\left[\theta(E_{\rm lev} - H + 0)(H + p^3\gamma^0\gamma^3)\right] - (H \to H_0)\,. \qquad (3.21)
\end{aligned}$$

At the last step we have omitted the terms which do not contribute to the trace owing to the hedgehog symmetry of the saddle point chiral field. The identity (3.19) allows us to ignore the $p^3\gamma^0\gamma^3$ piece. Comparing eq. (3.21) with the expression for the nucleon mass, eq. (3.17), we obtain finally:

$$M_2 \equiv \int\limits_0^1 dx\, x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)] = 1, \qquad (3.22)$$

meaning that quarks and antiquarks do carry the total momentum of the nucleon. The Pauli–Villars regularization is again privileged in that it does not destroy the momentum sum rule.

# 4 Isovector unpolarized distribution

## 4.1 Soliton rotation

It is easy to see that the isovector quark distribution function, $u(x) - d(x)$, vanishes in the leading order of the $1/N_c$ expansion. In order to obtain a nonvanishing result one has to consider rotational corrections to the classical soliton. In the leading order of the $1/N_c$ expansion the rotation of the soliton is taken into account by the rotational wave functions $\phi_{S_3T_3}(R)$ – see eq. (2.26). In higher orders in $1/N_c$ one has to take into account that the functional integral (1.1) goes over a time-dependent chiral field,

$$U(t, \mathbf{x}) = R(t) U_c(\mathbf{x}) R^\dagger(t) \tag{4.1}$$

For the rotating ansatz we have

$$i\partial_t - H(U) = R\,[i\partial_t - H(U_c) + iR^\dagger \dot{R}]\; R^\dagger . \tag{4.2}$$

This leads to the following modification of the leading-order eqs. (2.27, 2.34), which takes into account the time dependence of the Green function (2.32):

$$\begin{aligned}
&\langle \mathbf{P} = 0, S = T, S_3, T_3 | \psi^\dagger(x) \Gamma \psi(y) | \mathbf{P} = 0, S = T, S_3, T_3 \rangle \\
&= 2M_N i \int d^3\mathbf{X} \int dR\, \phi^\dagger_{S_3T_3}(R) \\
&\cdot \mathrm{Tr} \left\{ R^\dagger \Gamma R \langle y^0, \mathbf{y} - \mathbf{X} | [i\partial_t - H(U_c) + iR^\dagger \dot{R}]^{-1} | x^0, \mathbf{x} - \mathbf{X} \rangle \right\} \phi_{S_3T_3}(R),
\end{aligned} \tag{4.3}$$

where $\Gamma$ is an arbitrary matrix in spin and isospin. $\mathrm{Tr}\ldots$ denotes an ordinary trace in isospin and Lorentz indices, as contrasted to $\mathrm{Sp}\ldots$ standing for functional traces;

The integrand should be now expanded in the angular velocity $R^\dagger \dot{R}$ which should be replaced by the spin operator $S$ according to the following quantization rule [2, 6]:

$$R^\dagger \dot{R} \to \frac{i}{2I} S^a \tau^a , \tag{4.4}$$

where $I = O(N_c)$ is the moment of inertia of the soliton, see below, eq. (4.16). At large $N_c$ the moment of inertia is large, and the rotation may be considered as slow.

## 4.2 Expressing the isovector distribution through a double sum over levels

In the leading order in $1/N_c$ one can simply neglect $R^\dagger \dot{R}$ in eq. (4.3). In this approximation one reproduces eq. (3.1). However, neglecting the angular velocity altogether makes the nonsinglet distribution vanish. The first nonvanishing contribution to the isovector unpolarized quark distribution comes from the term linear in $R^\dagger \dot{R}$ in (4.3). Inserting this term into the general formula for the quark distribution function (2.15) we obtain

$$u(x) - d(x) = \frac{iM_N N_c}{4\pi} \sum_{S_3} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \int d^3\mathbf{X} \int dR\, \phi^\dagger_{S_3T_3}(R)$$

$$\cdot \mathrm{Tr}\Big\{ R^\dagger \tau^3 R (1+\gamma^0\gamma^3) \langle z^0, \mathbf{z}-\mathbf{X}| \frac{1}{i\partial_t - H(U_c)} (-iR^\dagger \dot{R})$$
$$\cdot \frac{1}{i\partial_t - H(U_c)} |0, -\mathbf{X}\rangle \Big\}\Big|_{z^3=-z^0,\, z_\perp=0} \phi_{S_3 T_3}(R)\,. \tag{4.5}$$

Applying the quantization rule (4.4) and introducing the orientation matrix in the adjoint representation,

$$D_{ab}(R) = \frac{1}{2}\mathrm{Tr}(R^\dagger \tau^a R \tau^b), \tag{4.6}$$

we find

$$u(x) - d(x) = \frac{N_c M_N i}{8\pi I} \sum_{S_3} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \int dR\, \phi^\dagger_{S_3T_3}(R) D_{3b}(R) S^a \phi_{S_3T_3}(R) \int d^3\mathbf{X}$$
$$\cdot \mathrm{Tr}\left\{ (\tau^b (1+\gamma^0\gamma^3) \langle z^0, \mathbf{z}-\mathbf{X}| \frac{1}{i\partial_t - H(U_c)}\, \tau^a \frac{1}{i\partial_t - H(U_c)} |0, -\mathbf{X}\rangle \right\}\Big|_{z^3=-z^0,\, z_\perp=0} \tag{4.7}$$

Let us first compute the rotational matrix element. Strictly speaking, this matrix element contains noncommuting operators $D_{3b}(R)$ and $S^a$, and one may worry about their ordering. However, due to the summing over nucleon spin $S_3$ the result does not depend on the order:

$$\sum_{S_3} \int dR\, \phi^\dagger_{S_3T_3}(R) D_{3b}(R) S^a \phi_{S_3T_3}(R)$$
$$= \sum_{S_3} \int dR\, \phi^\dagger_{S_3T_3}(R) S^a D_{3b}(R) \phi_{S_3T_3}(R) = -\frac{1}{3}\delta^{ab}(2T^3) \tag{4.8}$$

Therefore we get:

$$u(x) - d(x) = -(2T_3) \frac{N_c M_N i}{24\pi I} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \int d^3\mathbf{X} \sum_{a=1}^{3} \mathrm{Tr}\Big\{ \tau^a (1+\gamma^0\gamma^3)$$
$$\cdot \langle z^0, \mathbf{z}-\mathbf{X}| \frac{1}{i\partial_t - H(U_c)}\, \tau^a \frac{1}{i\partial_t - H(U_c)} |0, -\mathbf{X}\rangle \Big\}\Big|_{z^3=-z^0,\, z_\perp=0}\,. \tag{4.9}$$

Since $H(U_c)$ is time-independent (it coincides now with the hamiltonian $H$ of the previous sections) we rewrite the matrix element as

$$\langle z^0, \mathbf{z}-\mathbf{X}| \frac{1}{i\partial_t - H}\, \tau^a \frac{1}{i\partial_t - H} |0, -\mathbf{X}\rangle$$
$$= \int\limits_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega z^0} \langle \mathbf{z}-\mathbf{X}| \frac{1}{\omega - H}\, \tau^a \frac{1}{\omega - H} | -\mathbf{X}\rangle\,. \tag{4.10}$$

Noticing that $\langle \mathbf{z}-\mathbf{X}| = \langle -\mathbf{X}| \exp[i(\mathbf{p}\cdot\mathbf{z})]$ and integrating first over $\mathbf{X}$ and then over $z^0$ we get finally

$$u(x)-d(x)=-(2T_3)\frac{N_cM_Ni}{24\pi I}\int\limits_{-\infty}^{\infty}d\omega$$
$$\cdot\sum_{a=1}^{3}\mathrm{Sp}\left[\tau^a(1+\gamma^0\gamma^3)\delta(\omega+p^3-xM_N)\frac{1}{\omega-H}\,\tau^a\frac{1}{\omega-H}\right]. \qquad (4.11)$$

The result is of the same form as for the singlet structure function (3.8). However, in contrast to the singlet distribution which is a *single* sum over occupied (or non-occupied) levels (see eqs. (3.3, 3.4)), the isovector distribution is a *double* sum over levels. To see that explicitly we saturate the functional trace in eq. (4.11) by a complete set of functions, say, by the eigenfunctions of the Dirac hamiltonian $\Phi_n(\mathbf{p})$ in the momentum representation. Then the integration over $\omega$ is performed with the help of the $\delta$-function, and we get:

$$u(x)-d(x)=-(2T_3)\frac{N_cM_Ni}{24\pi I}\sum_{a=1}^{3}\sum_{m,n}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\Phi_n^\dagger(\mathbf{p})\tau^a(1+\gamma^0\gamma^3)\Phi_m(\mathbf{p})}{(p^3-xM_N-E_m)(p^3-xM_N-E_n)}$$
$$\cdot\int\frac{d^3\mathbf{p}'}{(2\pi)^3}\Phi_n^\dagger(\mathbf{p}')\tau^a\Phi_m(\mathbf{p}'). \qquad (4.12)$$

The denominators here should be in fact understood with an $i\epsilon$ prescription following from the original eq. (4.3):

$$\begin{aligned} &E_m\to E_m+i0 \qquad &&\text{for occupied states,}\\ &E_m\to E_m-i0 \qquad &&\text{for non-occupied states.}\end{aligned} \qquad (4.13)$$

We note that the wave functions $\Phi_n(\mathbf{p})$ generically have singularities in the complex plane, therefore one cannot, generally speaking, close the contour of integration in $p^3$ in eq. (4.12) and reduce the integral to a residue of one of the denominators in eq. (4.12) [2].

## 4.3 Isospin sum rule

Let us now check the isospin sum rule,

$$\int\limits_0^1 dx\{u(x)-d(x)-[\bar u(x)-\bar d(x)]\}\equiv\int\limits_{-1}^{1}dx[u(x)-d(x)]=2T_3. \qquad (4.14)$$

As usually for general relations, the derivation of this sum rule is more laconic when one uses the symbolic expressions with functional traces. Indeed, integrating eq. (4.11) over $x$ and replacing in the large $N_c$ limit the integration limits $[-1,1]$ by the whole real $x$ axis we obtain:

$$\int\limits_{-1}^{1}dx[u(x)-d(x)]=-(2T_3)\frac{iN_c}{12I}\int\frac{d\omega}{2\pi}\sum_{a=1}^{3}\mathrm{Sp}\Big(\tau^a(1+\gamma^0\gamma^3)\frac{1}{\omega-H}\,\tau^a\frac{1}{\omega-H}\Big). \quad (4.15)$$

[2] In that way one would recover eq.(25) from a recent paper [22]. We conclude that their eq.(25) does not agree with our result, eq. (4.12).

The hedgehog symmetry allows us to drop the term $\gamma^0\gamma^3$. Taking into account that the nucleon moment of inertia is [6]

$$I = -\frac{iN_c}{12}\int\frac{d\omega}{2\pi}\sum_{a=1}^{3}\mathrm{Sp}\Big(\tau^a\frac{1}{\omega-H}\,\tau^a\frac{1}{\omega-H}\Big), \tag{4.16}$$

we immediately reproduce the sum rule (4.14). One should keep in mind that both the moment of inertia and the isovector quark distribution contain logarithmic divergences, so that their regularization should be consistent with one another, if one wants to preserve the isospin sum rule.

## 4.4 Gottfried sum rule

The Gottfried sum rule [23] follows from the assumption that the antiquark distribution in the nucleon is isotopically invariant:

$$\int_0^1 dx[\bar{u}(x)-\bar{d}(x)] = -\int_{-1}^0 dx[u(x)-d(x)] = 0\,. \tag{4.17}$$

This relation does not follow from any fundamental principle of QCD. Nevertheless, certain models assume this symmetry. Experimentally, eq. (4.17) is violated rather strongly [24]. Let us see what does the chiral theory predict for the r.h.s. of eq. (4.17). Integrating eq. (4.11) over x from $-1$ to 0 we find:

$$\int_0^1 dx[\bar{u}(x)-\bar{d}(x)] = (2T_3)\frac{N_c i}{24\pi I}\int\frac{d\omega}{2\pi}\mathrm{Sp}\Big[\tau^a(1+\gamma^0\gamma^3)\theta(-\omega-p^3)\frac{1}{\omega-H}\tau^a\frac{1}{\omega-H}\Big]\,. \tag{4.18}$$

It can be checked that this quantity vanishes in the leading order of the gradient expansion, therefore the Gottfried sum rule is satisfied only in the limit of very large solitons. For real solitons the expression (4.18) is generally non-zero.

The Gottfried sum rule has been analyzed in the context of the chiral quark-soliton model by Wakamatsu [25]. Several suggestions for the r.h.s. of the sum rule have been considered in that paper, however none of them coincides literally with the exact result (4.18).

# 5 Isovector polarized distribution

The polarized quark distribution function (see e.g. [20]) is given by

$$\Delta q_f(x) = 2S_3 \frac{1}{4\pi} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \langle P, S|\psi_f^+(0)(1+\gamma^0\gamma^3)\gamma_5\psi_f(z)|P, S\rangle\Big|_{z^3=-z^0,\, z_\perp=0}\,, \tag{5.1}$$

where $S_3$ is the spin of the nucleon in its rest frame. At negative $x$ the function $\Delta q_f(x)$ has the meaning of the polarized antiquark distribution,

$$\Delta q_f(x) = \Delta \bar{q}_f(-x) \tag{5.2}$$

(note the opposite sign as compared to the relation (2.5) for the unpolarized distributions!)

One can easily check that in the leading order of the $1/N_c$ expansion only the *isovector* polarized distribution survives. We remind the reader that in the case of unpolarized distributions, on the contrary, the *singlet* distribution is large in $N_c$. Similarly to the derivation of eq. (3.3) from eq. (2.15) we can write

$$\begin{aligned}\Delta u(x) - \Delta d(x) &= (2S_3)\frac{N_c M_N}{2\pi} \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \int dR\, \phi^\dagger_{T_3 S_3}(R) \\ &\cdot \sum_{\substack{n \\ \text{occup.}}} e^{-iE_n z^0} \langle n|R^\dagger \tau^3 R(1+\gamma^0\gamma^3)\gamma_5 \exp(-iz^0 p^3)|n\rangle \phi_{T_3 S_3}(R)\,.\end{aligned} \tag{5.3}$$

Computing the rotational matrix element with the rotational wave functions $\phi(R)$ (see eq. (4.8)) we get:

$$\begin{aligned}\Delta u(x) - \Delta d(x) &= -(2T_3)\frac{N_c M_N}{6\pi} \\ &\cdot \int\limits_{-\infty}^{\infty} dz^0 e^{ixM_N z^0} \sum_{\substack{n \\ \text{occup.}}} e^{-iE_n z^0} \langle n|\tau^3(1+\gamma^0\gamma^3)\gamma_5 \exp(-iz^0 p^3)|n\rangle\,.\end{aligned} \tag{5.4}$$

One has to take here $T^3 = 1/2$ for the distribution functions in a proton and $T^3 = -1/2$ for those in a neutron. This sum over occupied states can be rewritten in terms of the functional trace:

$$\begin{aligned}\Delta u(x) - \Delta d(x) &= -\frac{1}{3}(2T_3)N_c M_N \\ &\cdot \int\limits_{-\infty}^{E_{\rm lev}+0} d\omega\ \mathrm{Sp}\left[\delta(\omega - H)\delta(\omega + p^3 - xM_N)\tau^3(1+\gamma^0\gamma^3)\gamma_5\right] - (H \to H_0).\end{aligned} \tag{5.5}$$

Integrating this equation over $x$ we reproduce the Bjorken sum rule:

$$\int_0^1 dx[\Delta u(x) - \Delta d(x) + \Delta\bar{u}(x) - \Delta\bar{d}(x)] = 2T_3\, g_A, \tag{5.6}$$

where $g_A$ is the nucleon axial constant. In deriving this sum rule one has to keep in mind that the nucleon axial constant $g_A$ in the leading order in $N_c$ is given by the following functional trace [26, 27]:

$$g_A = -\frac{N_c}{3}\int_{-\infty}^{E_{\rm lev}+0} d\omega\ \mathrm{Sp}\left[\delta(\omega - H)\tau^3\gamma^0\gamma^3\gamma_5\right] - (H\to H_0)\,. \tag{5.7}$$

It is understood that one has to calculate this trace with finite pion mass and only then go to the chiral limit.

# 6 Singlet polarized distribution

For this structure function one should replace $\tau^3$ in eq. (5.3) by a unity flavour matrix, hence $R^\dagger\tau^3 R$ is replaced by 1. For that reason the matrix element in eq. (5.3) is zero in the lowest order in the soliton rotation, and one has to expand the quark Green function to the first order in the angular velocity, $R^\dagger\dot{R}$, as for the isovector unpolarized distribution. Combining thus eq. (5.3) and eq. (4.5) we get

$$\begin{aligned}\Delta u(x) + \Delta d(x) &= \frac{iM_N N_c}{2\pi}(2S_3)\int_{-\infty}^{\infty} dz^0 e^{ixM_N z^0}\int d^3\mathbf{X}\int dR\,\phi^\dagger_{S_3T_3}(R)\\ &\cdot\mathrm{Tr}\Big((1+\gamma^0\gamma^3)\gamma_5\langle z^0, \mathbf{z}-\mathbf{X}|\frac{1}{i\partial_t - H}(-iR^\dagger\dot{R})\\ &\cdot\frac{1}{i\partial_t - H}|0,-\mathbf{X}\rangle\Big)\Big|_{z^3=-z^0,\, z_\perp=0}\phi_{S_3T_3}(R)\,.\end{aligned} \tag{6.1}$$

Making again the quantization substitution $R^\dagger\dot{R}\to iS^a\tau^a/(2I)$ and integrating over the soliton orientations $R$ we get a representation similar to eq. (4.9):

$$\begin{aligned}\Delta u(x) + \Delta d(x) &= \frac{iN_c M_N}{8\pi I}\int_{-\infty}^{\infty} dz^0 e^{ixM_N z^0}\int d^3\mathbf{X}\\ &\cdot\mathrm{Tr}\Big((1+\gamma^0\gamma^3)\gamma_5\langle z^0, \mathbf{z}-\mathbf{X}|\frac{1}{i\partial_t - H}\,\tau^3\frac{1}{i\partial_t - H}|0,-\mathbf{X}\rangle\Big)\Big|_{z^3=-z^0,\, z_\perp=0}.\end{aligned} \tag{6.2}$$

Performing the same steps as in subsection 4.2 we rewrite it through the functional trace:

$$\Delta u(x) + \Delta d(x) = \frac{iN_c M_N}{4I}\int\frac{d\omega}{2\pi}\mathrm{Sp}\Big(\gamma^0\gamma^3\gamma_5\delta(\omega + p^3 - xM_N)\frac{1}{\omega - H}\,\tau^3\frac{1}{\omega - H}\Big)\,, \tag{6.3}$$

which is similar in spirit to eq. (4.11). Again, repeating the derivation of subsection 4.2 we can write the singlet polarized distribution as a double sum over levels:

$$\Delta u(x) + \Delta d(x) = \frac{N_c M_N i}{8\pi I} \sum_{m,n} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\Phi_n^\dagger(\mathbf{p})(1+\gamma^0\gamma^3)\gamma_5 \Phi_m(\mathbf{p})}{(p^3 - xM_N - E_m)(p^3 - xM_N - E_n)}$$
$$\cdot \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \Phi_n^\dagger(\mathbf{p}')\tau^3 \Phi_m(\mathbf{p}')\,, \tag{6.4}$$

understood with the same $i\epsilon$ prescription as in eq. (4.13).

Integrating eq. (6.3) over $x$ we obtain the fraction of the nucleon spin carried by quarks:

$$\Delta u + \Delta d \equiv \int_{-1}^{1} dx[\Delta u(x) + \Delta d(x)] = \frac{iN_c}{4I} \int \frac{d\omega}{2\pi} \mathrm{Sp}\Big(\gamma^0\gamma^3\gamma_5 \frac{1}{\omega - H}\, \tau^3 \frac{1}{\omega - H}\Big) = g_A^{(0)}\,, \tag{6.5}$$

which coincides, as it should, with the expression for the nucleon singlet axial constant $g_A^{(0)}$ in the chiral quark-soliton model [9, 28]. The model calculation of this quantity gives $g_A^{(0)} \approx 0.36$ [28]; the rest of the nucleon spin (at low $q^2$!) is carried by the orbital moment of the constituents and of the distorted Dirac continuum [9]. At higher values of $q^2$ an increasing portion of the nucleon spin is carried by gluons.

# 7 Expressing distributions directly through the soliton field

All expressions for the quark distributions we have derived above are in fact certain functionals of the self-consistent pion field $U_c(\mathbf{x})$ which binds the nucleon. It would be helpful to write down these functionals in a more explicit form. That can be done but at a price of additional approximations which, however, appear to be rather accurate in practice.

A good place to start from is eq. (2.35) which relates structure functions to the imaginary part of the quark propagator in the background field of the soliton, $U_c(\mathbf{x})$. Expanding the propagator (2.32) in the derivatives of the pion field $\partial U_c$ up to the second order we obtain two contributions to the singlet distribution:

$$\sum_f q_f(x) \approx \frac{N_c M_N M^2}{2\pi} \,\mathrm{Im} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \frac{2\pi\delta(p^0 + p^3 - xM_N)}{(M^2 - (p-k)^2 - i0)(M^2 - p^2 - i0)^2}$$
$$\cdot \Big\{ (M^2 - p^2)\ \mathrm{Tr}\left[\tilde{U}^{-\gamma_5}(-\mathbf{k}) \not{k} \tilde{U}^{-\gamma_5}(\mathbf{k})(\gamma^0 + \gamma^3)\right]$$
$$+ \mathrm{Tr}\left[\not{p}\not{k}\tilde{U}^{-\gamma_5}(-\mathbf{k})\not{k}\tilde{U}^{-\gamma_5}(\mathbf{k})(\gamma^0 + \gamma^3)\right]\Big\}, \tag{7.1}$$

where we have introduced the Fourier transform of the pion field,

$$\tilde{U}(\mathbf{k}) = \int d^3\mathbf{r}\, e^{-i(\mathbf{kr})}\, [U_c(\mathbf{r}) - 1]. \tag{7.2}$$

The integration over the quark loop momenta $p$ can be easily performed: one first uses the $\delta$-function to integrate over $p^0$, then one integrates over $p^3$ taking the residues of one of the denominators. The condition that the poles in $p^3$ lie on different sides of the integration axis so that one gets a non-zero imaginary part, is $k^3 > xM_N$ for $x > 0$ and $k^3 < xM_N$ for $x < 0$. For $x < 0$ we change the dumb variable $\mathbf{k} \to -\mathbf{k}$, so that the condition can be written in a common form as $k^3 > |x|M_N$. We get

$$\sum_f q_f(x) \approx \mathrm{sign}(x) \frac{N_c M_N M^2}{\pi} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\ \mathrm{Tr}\ \left\{ \tilde{U}(\mathbf{k}) [\tilde{U}(\mathbf{k})]^\dagger \right\} \theta(k^3 - |x|M_N)$$
$$\cdot \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \frac{M^2 + \mathbf{p}_\perp^2}{(M^2 + \mathbf{p}_\perp^2 + \kappa^2)^2}, \qquad \kappa^2 = \frac{|x|M_N(k^3 - |x|M_N)\mathbf{k}^2}{(k^3)^2} \ > \ 0. \tag{7.3}$$

The last integral is logarithmically divergent. We regularize it by the Pauli–Villars method: one has to compute the structure functions replacing the quark mass $M \to M_{PV}$ where $M_{PV}$ is the Pauli–Villars regulator mass, multiply the obtained result by $M^2/M_{PV}^2$ and subtract it from the original distribution. At the moment it is the only practical way of regularizing the structure functions we know of, which does not violate casuality and analyticity, see above. Note that in calculating static characteristics of the nucleon the requirements on the regularization method are not so restrictive. The value of the Pauli–Villars mass is fixed from the value of the $F_\pi$ constant [6]:

$$F_\pi^2 = 4N_c \int \frac{d^4k}{(2\pi)^4}\ \frac{M^2}{(M^2 + k^2)^2} - 4N_c \frac{M^2}{M_{PV}^2} \int \frac{d^4k}{(2\pi)^4}\ \frac{M_{PV}^2}{(M_{PV}^2 + k^2)^2} = \frac{N_c M^2}{4\pi^2} \ln \frac{M_{PV}^2}{M^2}. \tag{7.4}$$

Integrating eq. (7.3) over $\mathbf{p}_\perp$ and performing the Pauli-Villars regularization we obtain finally for the singlet structure function:

$$\sum_f q_f(x) \approx \mathrm{sign}(x) \frac{N_c M_N M^2}{4\pi^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \theta(k^3 - |x| M_N) \ \mathrm{Tr}\ \left( \tilde{U}(\mathbf{k}) [\tilde{U}(\mathbf{k})]^\dagger \right)$$
$$\cdot \left[ \ln \frac{M_{PV}^2 + \kappa^2}{M^2 + \kappa^2} - \frac{\kappa^2 (M_{PV}^2 - M^2)}{(M_{PV}^2 + \kappa^2)(M^2 + \kappa^2)} \right]. \tag{7.5}$$

We call it *interpolation formula* as eq. (7.5) becomes exact in three limiting cases: i) low momenta, $|\partial U| \ll M$, ii) high momenta, $|\partial U| \gg M$, iii) any momenta but small pion fields, $|\log U| \ll 1$. Therefore, we expect that the interpolation formula has a good accuracy also in a general case. As compared to exact calculations involving summation over all levels the use of the interpolation formula gives an enormous gain in computing time: for a given profile of the pion field in the nucleon one has to compute numerically just three integrals.

If the spatial size of the soliton is large, meaning that momenta $\mathbf{k}$ in eq. (7.5) are small, one can neglect $\kappa^2$ in the quark loop integral and get a simple formula:

$$u(x) + d(x) \approx \mathrm{sign}(x) F_\pi^2 M_N \int \frac{d^3\mathbf{k}}{(2\pi)^3} \theta(k^3 - M_N |x|) \ \mathrm{Tr} \left( \tilde{U}(\mathbf{k}) [\tilde{U}(\mathbf{k})]^\dagger \right) \tag{7.6}$$

This is a remarkable equation. First, it shows that if the nucleon has the size $r_0$ the singlet structure function is concentrated around the values of $x \sim 1/(r_0 M_N)$. Second, we notice that eq. (7.6) is *non-analytic* in the pion field momenta. It means that one cannot make a gradient expansion either for the structure functions or for their moments. This situation is new: all static characteristics of the nucleon admit expansion in the gradients of the pion field [6]. By the way, it means that there is no reasonable way to extract structure functions from the Skyrme model (which contains just two and four derivatives of the pion field), even if one invents a way to identify leading-twist operators in that model.

It is amusing that the second moment of the distribution (7.6) is $2/3$ instead of unity. Indeed, integrating eq. (7.6) over $x$ with $x$ one gets for the second moment of the singlet structure function

$$M_2 \approx \frac{F_\pi^2}{M_N} \int\limits_{k^3 > 0} \frac{d^3\mathbf{k}}{(2\pi)^3} \ k_3^2 \ \mathrm{Tr} \left( \tilde{U}(\mathbf{k}) [\tilde{U}(\mathbf{k})]^\dagger \right) \tag{7.7}$$

where $k_3^2$ can be replaced by $\mathbf{k}^2/3$ for the spherically symmetric hedgehog field. At the same time the leading contribution to the large-size nucleon mass is given just by the kinetic-energy term of the chiral lagrangian,

$$M_N \approx \frac{F_\pi^2}{4} \int d^3\mathbf{x} \ \mathrm{Tr} \ [\partial_i U(\mathbf{x}) \partial_i U^\dagger(\mathbf{x})]. \tag{7.8}$$

Comparing eqs. (7.7, 7.8) we see that in the large-size approximation the nucleon energy at rest computed from the total energy carried by its constituents is $2/3$ that of its mass! Exactly this "2/3" paradox has been discovered some time ago in a similar situation by Lorentz [29] who attempted to calculate the energy of the electron from a

charge distribution bound by some unknown forces of non-electromagnetic origin. He found an interesting relation: $E = \frac{2}{3}mc^2$. The paradox is resolved when one exploits the equation of motion for constituents. In our case eq. (7.8) has no minimum, however the full functional for the nucleon mass, including the discrete level, has. As shown in subsection 3.5 the use of the equation of motion for the pion field is essential in establishing the correct momentum sum rule.

Eq. (7.6) shows that in the leading order of the above expansion the singlet quark distribution coincides with the antiquark one. In order to obtain a nonvanishing result for the difference of singlet quark and antiquark distributions one has to go to the next (third) order in expanding the quark propagator. This time we perform the expansion in the coordinate space and get an elegant expression:

$$\sum_f [q_f(x) - \bar{q}_f(x)] \approx \frac{N_c M_N}{4\pi^3} \int d^3\mathbf{y} \int_{-\infty}^{\infty} d\xi\, e^{-i\xi M_N x} \int_0^1 d\alpha \int_0^\alpha d\beta \int_0^\beta d\gamma$$
$$\cdot \epsilon_{ijk} \mathrm{Tr}\left( [\partial_i U(\mathbf{y} + \alpha\xi\mathbf{e}_3)][\partial_j U(\mathbf{y} + \beta\xi\mathbf{e}_3)]^\dagger [\partial_k U(\mathbf{y} + \gamma\xi\mathbf{e}_3)][U(\mathbf{y})]^\dagger \right). \quad (7.9)$$

It is remarkable that this result is consistent with the baryon number sum rule (3.12). Indeed, for the solitons of a large size (for which the above expansion is justified) the baryon number coincides [6] with the winding number of the pion field:

$$B = \frac{1}{24\pi^2} \int d^3y \epsilon_{ijk} \mathrm{Tr}\left[ (U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U) \right]. \quad (7.10)$$

Integrating eq. (7.9) over $x$ one immediately obtains

$$\int_0^1 dx \sum_f [q_f(x) - \bar{q}_f(x)] = N_c B \quad (7.11)$$

where $B$ is given by the soliton winding number, eq. (7.10).

Turning to the polarized distributions we remind the reader that in the leading order of the $1/N_c$ expansion only the isovector function $\Delta u(x) - \Delta d(x)$ survives. Similarly to eq. (7.5) we obtain:

$$\Delta u(x) - \Delta d(x) \approx \frac{1}{3}\,(2T_3)\frac{N_c M_N M^2}{4\pi^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \theta(k^3 - |x| M_N)\ \mathrm{Tr}\ \left( \tilde{U}(\mathbf{k})[\tilde{U}(\mathbf{k})]^\dagger \tau^3 \right)$$
$$\cdot \left[ \ln \frac{M_{PV}^2 + \kappa^2}{M^2 + \kappa^2} - \frac{\kappa^2 (M_{PV}^2 - M^2)}{(M_{PV}^2 + \kappa^2)(M^2 + \kappa^2)} \right], \quad (7.12)$$

where $\kappa^2$ is given in eq. (7.3). For large-size solitons it can be simplified to

$$\Delta u(x) - \Delta d(x) \approx \frac{1}{3}\,(2T_3) F_\pi^2 M_N \int \frac{d^3k}{(2\pi)^3}\, \theta(k^3 - M_N|x|) \mathrm{Tr}\left( U(\mathbf{k})[U(\mathbf{k})]^\dagger \tau^3 \right). \quad (7.13)$$

Note that eqs. (7.12, 7.13) are even in $x$; the odd part of the Dirac continuum contribution to $\Delta u(x) - \Delta d(x)$ arises in the next order in the interpolation formula and is thus small.

Equations (7.5, 7.9, 7.12) can be used to make a quick estimate of the structure functions, before one plugs into exact numerics which, in any case, is rather laborious. One should not forget, however, to add the contribution of the discrete level. To be consistent, a contribution of the discrete level with a Pauli–Villars mass to a distribution function $f(x)$ should be subtracted:

$$f^{\rm lev}(x) \to f^{\rm lev}_M(x) - \frac{M^2}{M_{PV}^2} f^{\rm lev}_{M_{PV}}(x). \tag{7.14}$$

# 8 Numerical results and discussion

We have calculated numerically two structure functions surviving in the leading order in $N_c$: the singlet unpolarized distribution (section 3) and the isovector polarized one (section 5); in both cases we get quark and antiquark distributions separately.

A variational estimate of the best profile of the pion-field soliton (see eq. (2.22)) has been performed in ref. [6] yielding for $M = 350$ MeV

$$P(r) = -2\,\mathrm{arctg}\left(\frac{r_0^2}{r^2}\right), \qquad r_0 \approx 1.0/M, \qquad M_N \approx 1170\ MeV. \tag{8.1}$$

This profile function has a correct behaviour at small and large distances and is stable in respect to small perturbations. In our numerics we use the analytical profile (8.1): its difference with the exact solution is small. The value of the Pauli-Villars mass needed for the regularization is found from eq. (7.4) to be $M_{PV} \approx 560\ MeV$.

We estimate the Dirac continuum contribution using the interpolation formulae: eq. (7.5) for the unpolarized singlet distribution and eq. (7.12) for the isovector polarized distribution. In fact we have performed the exact computation of the Dirac continuum contribution by direct summation over all levels: as illustrated by Figs.1,2, its deviation from the interpolation formula proves to be small. The description of exact computations will be a subject of a separate publication [30]. Since the reader may wish to repeat the calculations with his or her favourite set of parameters, we suggest the use of the interpolation formulae which give reliable approximations to the structure functions but can be computed in a few minutes on a PC.

As stressed many times in the paper, the quark distribution $q(x)$ is a single function defined for both positive and negative $x$'s. At $x < 0$ it gives actually the distribution of antiquarks, $\bar{q}(x) = -q(-x)$. The $x$-even combination, that is $q(x) - \bar{q}(x),\ x > 0$, which is actually the baryon number density, is finite when one takes the ultraviolet cutoff to infinity. Moreover, it *should not* be regularized at all as it corresponds to the imaginary part of the effective chiral action (1.1). Probably, an ultraviolet regularization can be introduced which takes into account automatically the non-renormalization of the imaginary part of the chiral action. The original regularization by the momentum-dependent constituent quark mass $M(p)$ [4] seems to satisfy this requirement [31]. In this paper, however, we mimic that regularization *i)* by the relativistic Pauli–Villars regularization for the logarithmically divergent $x$-odd part of $q(x)$, that is for $q(x) + \bar{q}(x)$, *ii)* by making no regularization for the $q(x) - \bar{q}(x)$ distribution. In the case of isovector polarized distributions, the logarithmically divergent combination is $[\Delta u(x) - \Delta d(x)] + [\Delta\bar{u}(x) - \Delta\bar{d}(x)],\ x > 0$, corresponding to the $g_1^p - g_1^n$ structure function; the other combination originates from the imaginary part of the effective chiral action and should not be regularized.

Figures 1–5 present our results for the following distribution functions:

- Fig.1: $x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]/2$;
- Fig.2: $x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]/2$;
- Fig.3: $x[\bar{u}(x) + \bar{d}(x)]/2$;
- Fig.4: $x[\Delta u(x) - \Delta d(x) + \Delta\bar{u}(x) - \Delta\bar{d}(x)]/2$;
- Fig.5: $x[\Delta\bar{u}(x) - \Delta\bar{d}(x)]/2$.

In Figs. 1 and 4 we plot the contributions of the discrete level and that of the Dirac continuum (computed via the interpolation formula) as well as their sum separately. The exact calculation of the sum is also shown in Fig.1. It can be seen that the interpolation formula gives a good approximation to the exact calculations. The same is seen from Fig.2. The discrete-level contributions are given in Appendix B. One can see that our quark and antiquark distribution functions are positive (Fig.3), and that the contribution of the Dirac continuum is significant.

It should be emphasized that the distribution of *antiquarks* arising from the discrete level (see eq. (B.5)) is definitely negative (and sizeable!) [3] Positivity of the structure functions is restored only when one includes the contribution of the Dirac continuum. Indeed, the existence of a non-trivial discrete level is due to a strong mean pion field in the large-$N_c$ nucleon. Taking into account only the discrete-level contribution to the structure functions means ignoring the degrees of freedom stored in the field creating that level, hence the "negative probability", $\bar{q}(x) < 0$. Adding the Dirac continuum contribution makes the system complete, and all the probabilities become positive. We have demonstrated above that all the general sum rules are fulfilled only when one adds the discrete level and the Dirac continuum contributions together.

There seems to be a lesson here for all variants of bag models. It is usually assumed that the three quarks in the bag give rise only to quark distributions, however they inevitably produce also *negative* antiquark distributions. The baryon number computed from the quark distributions only, turns out to be *less* than unity; to restore unity one has to subtract the negative distribution of the antiquarks. This circumstance is not often emphasized. To cure this decease one would need to add the structure function arising from the forces which keep the three quarks bound, in this case from the bag surface.

To stress it once again: the discrete level produces not only valence quarks (in the sense used in the deep inelastic scattering phenomenology), whereas the Dirac continuum produces non-equal distributions of quarks and antiquarks (see Fig.3), therefore its contribution should not be identified with the quark-antiquark sea of the DIS folklore.

In the large $N_c$ limit the nucleon is heavy, so we have neglected its recoil. For that reason the structure functions do not automatically go to zero at $x = 1$. However at $x \gg 1/N_c$ the distributions behave as $\sim \exp(-\text{const} \cdot N_c x)$; numerically, even for $N_c = 3$ all distributions computed are very small at $x \approx 1$. Nevertheless, we should caution those who may wish to reconstruct the structure functions from the moments: at $n \geq N_c$ the $n$-th moments become sensitive to the tail of the structure functions at $x \sim 1$ where $1/N_c$ corrections become 100% important, and the moments become thus absolutely unreliable.

[3] In the extreme case of a very strongly bound discrete level when it approaches the lower continuum, this level would not produce quarks at all – only antiquarks, but with a negative sign!

We remind the reader that our calculations refer to the leading-twist distribution functions at a normalization point of about 600 MeV. In order to make a meaningful comparison with the data one has to use our distributions as initial conditions for the standard perturbative evolution of the structure functions to higher values of $q^2$ where the actual data are available. This evolution takes into account the bremsstrahlung of gluons and also their conversion into quark-antiquark pairs. It is well known that the perturbative evolution makes the distributions more "soft". Therefore, it can well wash out the quark excess at high $x$ and cure the deficit at low $x$. This part of the investigation remains to be done.

However, we can still make a comparison if not directly with the data, then with the parametrization of the data at a low normalization point $\mu \approx 600\ MeV$ performed recently by Glück, Reya *et al.* [15, 17]. The standard perturbative evolution of their distributions describes well all the existing data at larger values of $q^2$. It is known, though, that the evolution in the opposite direction is highly unstable. However, we believe that the low-point distributions suggested in refs. [15, 17] are reasonable, and we compare our curves to their in Figs. 1–4.

Despite using the parametrization of the data at a low value of $q^2$, the "experimental" distributions appear to be more "soft" than the calculated ones. On the whole it looks as if we have determined the distributions at an even lower normalization point than that of refs. [15, 17]. Since $\alpha_s$ at these momenta are large it may require quite a short evolution range to move our distributions to those of [15, 17].

The calculated isovector polarized distribution shown in Fig.3 appears to be systematically less than the parametrization of ref. [17]. We would like to make three comments on this discrepancy. First, we think that the parametrization of the polarized distributions is less reliable than that of the unpolarized ones as it is based on less data with larger errors. In particular, the authors of [17] have assumed that $\Delta\bar{u}(x) - \Delta\bar{d}(x) = 0$ which is not confirmed in the model we are considering: this quantity appears to be of the same order as $\Delta\bar{u}(x) + \Delta\bar{d}(x)$ of ref. [17], see Fig.5. Second, by choosing the parameters of the soliton in eq. (8.1) we have, unfortunately, implanted a somewhat small value of the $g_A$ constant to which the distribution of Fig. 4 is normalized according to the Bjorken sum rule: it is $g_A \approx 0.96$ instead of 1.25. Third, it is known [9, 10] that this channel is particularly sensitive to the $1/N_c$ corrections which are altogether neglected in this paper.

On the whole we get a reasonable description of several parton distributions without adjusting the parameters of the model to make a "best fit".

Finally, we would like to comment on a recent work [22] where quark distributions have been estimated in the Nambu–Jona-Lasinio model which, after certain simplifications, is reduced to the chiral quark-soliton model of ref.[6] considered here. Only the contribution of the discrete level to the unpolarized structure functions has been studied in that work. As explained above, this approximation leads to a number of inconsistencies. For example, one obtains a wrong (negative) sign for the singlet antiquark distribution function, while the baryon number obtained from integrating the quark distribution only, is less than unity. As to the isovector distributions also considered in [22], their basic eq. (25) does not seem to agree with our result given by eq. (4.12).

# 9 Conclusions

At large number of colours the nucleon can be viewed as a heavy semiclassical body whose $N_c$ "valence" quarks are bound by a self-consistent pion field. The energy of the pion field is given by the effective chiral lagrangian and coincides with the aggregate energy of the Dirac sea of quarks (the free continuum subtracted). Therefore, to compute the deep-inelastic structure functions in the large $N_c$ limit, it is sufficient to calculate the quark and antiquark distributions arising from the discrete level occupied by quarks, and from the (distorted) negative-energy Dirac continuum. In contrast to all variants of the bag model, the completeness of the states involved guarantees the consistency of the calculations. Indeed, we have checked the validity of the baryon number, isospin, total momentum and of the Bjorken sum rules. We have also derived an expression for the r.h.s. of the Gottfried sum rule, which differs from the previously suggested ones. To our knowledge, it is the first time that the nucleon structure functions are theoretically calculated in a relativistic model which preserves all general properties of parton distributions.

In the academic limit of a very weak mean pion field the Dirac continuum reduces to the free one (and should be subtracted to zero) while the discrete level joins the upper Dirac continuum. In that limit there are no antiquarks, while the distribution function of quarks becomes $\delta(x-1/N_c)$. In reality there is a non-trivial mean pion field which *i*) creates a discrete level, *ii*) distorts the negative-energy Dirac continuum. As a result, the above $\delta$-function is smeared significantly (but still in the range of the order of $1/N_c$), and non-zero antiquark distributions appear even at a low normalization point. It should be stressed that antiquarks come not only from the Dirac continuum but also from the discrete level, whereas the Dirac continuum produces nonequal distributions of quarks and antiquarks.

As to the gluon distribution, it depends on the details of the ultraviolet regularization of the effective chiral theory (corresponding to the "formfactor" of the constituent quark), and for that reason we have not attempted to determine it here. Also, we have not tried to make a "best fit" to the parametrizations of the data [15, 17] by adjusting the parameters of the model, such as the constituent quark mass and the way one regularizes the effective chiral theory. In fact we have used practically the same set of parameters as deduced [4, 6] from instantons. It is remarkable that we are able to reproduce the basic features of the distributions, despite neglecting completely the $1/N_c$ corrections. It would certainly be preferable to use our leading-twist distributions as initial conditions to the standard perturbative evolution to higher values of $q^2$ where a direct comparison to the data is possible, instead of comparing to the parametrizations of the data at a low normalization point, which to some extent is model-dependent.

We have shown that, from the point of view of large $N_c$, all distributions can be divided into "large" and "small" ones. The large ones are the singlet unpolarized quark and antiquark distributions and the isovector polarized distributions. The isovector unpolarized and the singlet polarized are, parametrically, $N_c$ times smaller, which seems to be confirmed experimentally despite that in reality $N_c$ is only three.

We have found that the structure functions are non-analytic in the pion field momenta, though one can still write exact expressions for the structure functions in the limit of large-size nucleons, see section 7. For arbitrary sizes we have derived *interpolation formulae* for the structure functions which allow one to compute the structure functions in a few minutes on a PC and reproduce the results of exact calculations to

good accuracy.

We have restricted ourselves to the case of $u$ and $d$ quarks only, though the generalization to three flavours is quite simple in the collective-quantization technique – see, e.g., ref. [32].

The methods developed in this paper can be easily generalized to higher-twist observables, like the $g_2, h_T$... structure functions. They can be also used to estimate power corrections to the numerous structure functions at relatively low $q^2$.

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# A Expressing distributions through the Feynman propagator

The Feynman Green function in a stationary background field is defined as

$$\begin{aligned}
&G_{ij}(x,y) = i\langle 0|\mathrm{T}\left\{\psi_i(y)\bar{\psi}_j(x)\right\}|0\rangle \\
&= i\theta(x^0-y^0) \sum_{\substack{n \\ \mathrm{non-occup.}}} \exp[-iE_n(x^0-y^0)]\Phi_{n,i}(\mathbf{x})\bar{\Phi}_{n,j}(\mathbf{y}) \\
&-i\theta(y^0-x^0) \sum_{\substack{n \\ \mathrm{occup.}}} \exp[-iE_n(x^0-y^0)]\Phi_{n,i}(\mathbf{x})\bar{\Phi}_{n,j}(\mathbf{y})\,.
\end{aligned} \tag{A.1}$$

In the free case (no background field) it comes to

$$G_{ij}(x,y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-i(p\cdot(x-y))}\, \frac{M+\not{p}}{M^2-p^2-i0}. \tag{A.2}$$

The sign convention is such that the imaginary part of the Feynman propagator is positive.

Let us rewrite the singlet structure function, say, in representation of eq. (3.1) through the Feynman propagator. To that end we divide the full range of integration in $z^0$ in eq. (3.1) in two parts: from 0 to $+\infty$ and from $-\infty$ to 0. Let us call these two contributions to the full structure function $q_1(x)$ and $q_2(x)$ respectively. In the $q_2(x)$ part let us change the integration variable $z^0 \to -z^0$, so that the new variable also runs from 0 to $+\infty$. We have from eq. (3.1):

$$q_1(x) = \frac{N_c M_N}{2\pi} \int d^3\mathbf{X} \int_0^\infty e^{ixM_N z^0} \sum_{\substack{n \\ \mathrm{occup.}}} e^{-iE_n z^0} \bar{\Phi}_n(-\mathbf{X})(\gamma^0+\gamma^3)\Phi_n(-z^0\mathbf{n}_3-\mathbf{X})\,, \tag{A.3}$$

$$q_2(x) = \frac{N_c M_N}{2\pi} \int d^3\mathbf{X} \int_0^\infty e^{-ixM_N z^0} \sum_{\substack{n \\ \mathrm{occup.}}} e^{iE_n z^0} \bar{\Phi}_n(-\mathbf{X})(\gamma^0+\gamma^3)\Phi_n(z^0\mathbf{n}_3-\mathbf{X})\,. \tag{A.4}$$

Comparing eq. (A.4) with the definition of the Feynman Green function (A.1) we see that it can be written as

$$q_2(x) = i\frac{N_c M_N}{2\pi} \int d^3\mathbf{X} \int_0^\infty e^{-ixM_N z^0}\ \mathrm{Tr}\ G(z^0\mathbf{n}_3-\mathbf{X},-z^0;\ -\mathbf{X},0)(\gamma^0+\gamma^3). \tag{A.5}$$

The other part, eq. (A.3), does not directly fit into the definition of the Feynman Green function, however its complex conjugate does. Indeed, making the complex conjugation of eq. (A.3) and changing the integration variable $\mathbf{X} \to \mathbf{X} - z^0\mathbf{n}_3$ we get exactly the r.h.s. of eq. (A.4), therefore $q_1^*(x) = q_2(x)$. Using eq. (A.5) we get for the full singlet structure function

$$\sum_f q_f(x) = -2\ \mathrm{Im}\ \frac{N_c M_N}{2\pi} \int d^3\mathbf{X} \int_0^\infty e^{-ixM_N z^0}\ \mathrm{Tr}\ G(z^0\mathbf{n}_3 - \mathbf{X}, -z^0;\ -\mathbf{X}, 0)(\gamma^0 + \gamma^3). \tag{A.6}$$

Let us now pass to the Fourier transform of the propagator. For the time-independent background field one writes:

$$G(x,y) = \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4}\ 2\pi\delta(p_1^0 - p_2^0) S(p_1^0, \mathbf{p}_1, \mathbf{p}_2) e^{i(p_1\cdot x) - i(p_2\cdot y)}. \tag{A.7}$$

Putting it in eq. (A.6) we integrate over $\mathbf{X}$, which yields $(2\pi)^3\delta(\mathbf{p}_1 - \mathbf{p}_2)$. Finally, we integrate eq. (A.6) in $z^0$ and obtain

$$\sum_f q_f(x) = 2\ \mathrm{Im}\ \frac{N_c M_N}{2\pi} \int \frac{d^4p}{(2\pi)^4 i} \frac{\mathrm{Tr}[S(p^0, \mathbf{p}, \mathbf{p})(\gamma^0 + \gamma^3)]}{p^0 + p^3 - xM_N + i0}. \tag{A.8}$$

We have made this derivation starting from eq. (3.1) where the summation over occupied levels is used. We could as well start from the equivalent eq. (3.4) where summation goes over non-occupied levels. Repeating the same steps as above we arrive to eq. (A.8) but with the opposite overall sign and the opposite sign of $i0$ in the denominator. Since the two formulas must be equivalent in a "good" renormalization scheme, it means that a non-zero imaginary part of the whole expression arises solely from the $i\pi\delta(p^0 + p^3 - xM_N)$ pieces of the denominators in both cases. Thus, we obtain:

$$\sum_f q_f(x) = -\ \mathrm{Im}\ \frac{N_c M_N}{2\pi} \int \frac{d^4p}{(2\pi)^4}\ 2\pi\delta(p^0 + p^3 - xM_N)\ \mathrm{Tr}\ [S(p^0, \mathbf{p}, \mathbf{p})(\gamma^0 + \gamma^3)], \tag{A.9}$$

which is eq. (2.35) of the main text.

# B Bound-state level

We present here the contributions of the discrete bound-state level to the singlet unpolarized and to the isovector polarized structure functions. This is the cases where the discrete-level contribution is well-defined and in fact large. The other two structure functions considered in the paper are expressed through a *double* sum over levels, hence the contribution of the discrete level is not specific.

The bound-state level occurs in the grand spin $K = 0$ and parity $\Pi = +$ sector of the Dirac hamiltonian (2.19). In that sector the eigenvalue equation takes the form [4]:

$$\begin{pmatrix} M\cos P(r) & -\frac{\partial}{\partial r} - \frac{2}{r} - M\sin P(r) \\ \frac{\partial}{\partial r} - M\sin P(r) & -M\cos P(r) \end{pmatrix} \begin{pmatrix} h_0(r) \\ j_1(r) \end{pmatrix} = E_{\mathrm{lev}} \begin{pmatrix} h_0(r) \\ j_1(r) \end{pmatrix}. \tag{B.1}$$

We assume that the radial wave functions are normalized by the condition

[4] We change the sign of the $\gamma_5$ matrix and hence of the profile function $P(r)$ as compared to ref. [6]. The $\gamma_5$ matrix is now that of Bjorken and Drell. The profile function is equal to $-\pi$ at the origin.

$$\int_0^\infty dr\, r^2[h_0^2(r)+j_1^2(r)]=1. \tag{B.2}$$

We introduce the Fourier transforms of the radial wave functions,

$$h(k)=\int_0^\infty dr\, r^2\, h_0(r)R_{k0}(r), \qquad j(k)=\int_0^\infty dr\, r^2\, j_1(r)R_{k1}(r), \tag{B.3}$$

where

$$R_{kl}(r)=\sqrt{\frac{k}{r}}J_{l+\frac{1}{2}}(kr)=(-1)^l\sqrt{\frac{2}{\pi}}\frac{r^l}{k^l}\left(\frac{1}{r}\frac{d}{dr}\right)^l\frac{\sin kr}{r}. \tag{B.4}$$

The bound-state level contribution to the singlet unpolarized structure function can be simply obtained from the general eq. (3.3). We get:

$$[u(x)+d(x)]_{\rm val}(x)=N_cM_N\int\limits_{|xM_N-E_{\rm lev}|}^\infty\frac{dk}{2k}\left\{h^2(k)+j^2(k)-2\frac{xM_N-E_{\rm lev}}{k}h(k)j(k)\right\}. \tag{B.5}$$

Note that the r.h.s. is positive for all values of $x$, in particular at $x<0$ where eq. (B.5) determines in fact the antiquark distribution. Since $\bar{q}(x)=-q(-x)$, it means that eq. (B.5) gives a *negative* distribution of antiquarks at $x>0$. At the same time it is easy to check by integrating eq. (B.5) that the baryon number sum rule is fully saturated by the discrete-level contribution only. It means that without the subtraction of the *negative* antiquark distribution the baryon number is not conserved. Simultaneously it means that the total baryon number of the Dirac continuum is zero, though locally in $x$ the antiquark distribution from the Dirac continuum does not necessarily coincide with the quark one, see the dotted line in Fig.2.

The bound-state contribution to the polarized isovector distribution function in the proton is obtained from eq. (5.4):

$$[\Delta u(x)-\Delta d(x)]_{\rm val}=\frac{1}{3}N_cM_N\int\limits_{|xM_N-E_{\rm lev}|}^\infty\frac{dk}{2k}\left\{h^2(k)+\left[2\frac{(xM_N-E_{\rm lev})^2}{k^2}-1\right]j^2(k)\right.$$
$$\left.-2\frac{(xM_N-E_{\rm lev})}{k}h(k)j(k)\right\}. \tag{B.6}$$

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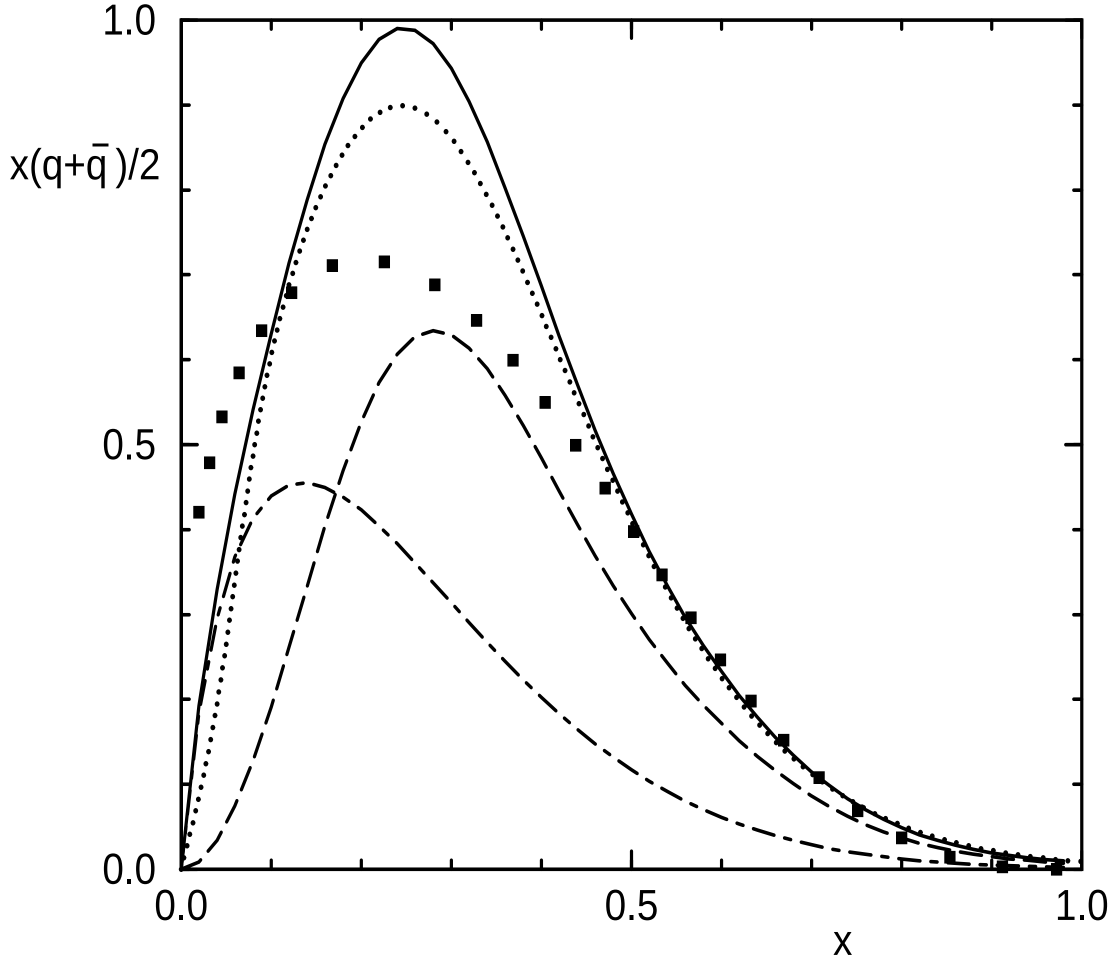


Figure 1: The singlet unpolarized distribution, $x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]/2$. Dashed line: regularized contribution from the discrete level; dash-dotted line: contribution from the Dirac continuum according to the interpolation formula, eq. (7.5); solid line: the total distribution being the sum of the dashed and dash-dotted curves, dotted line: the exact total distribution; squares: the parametrization of ref. [15].

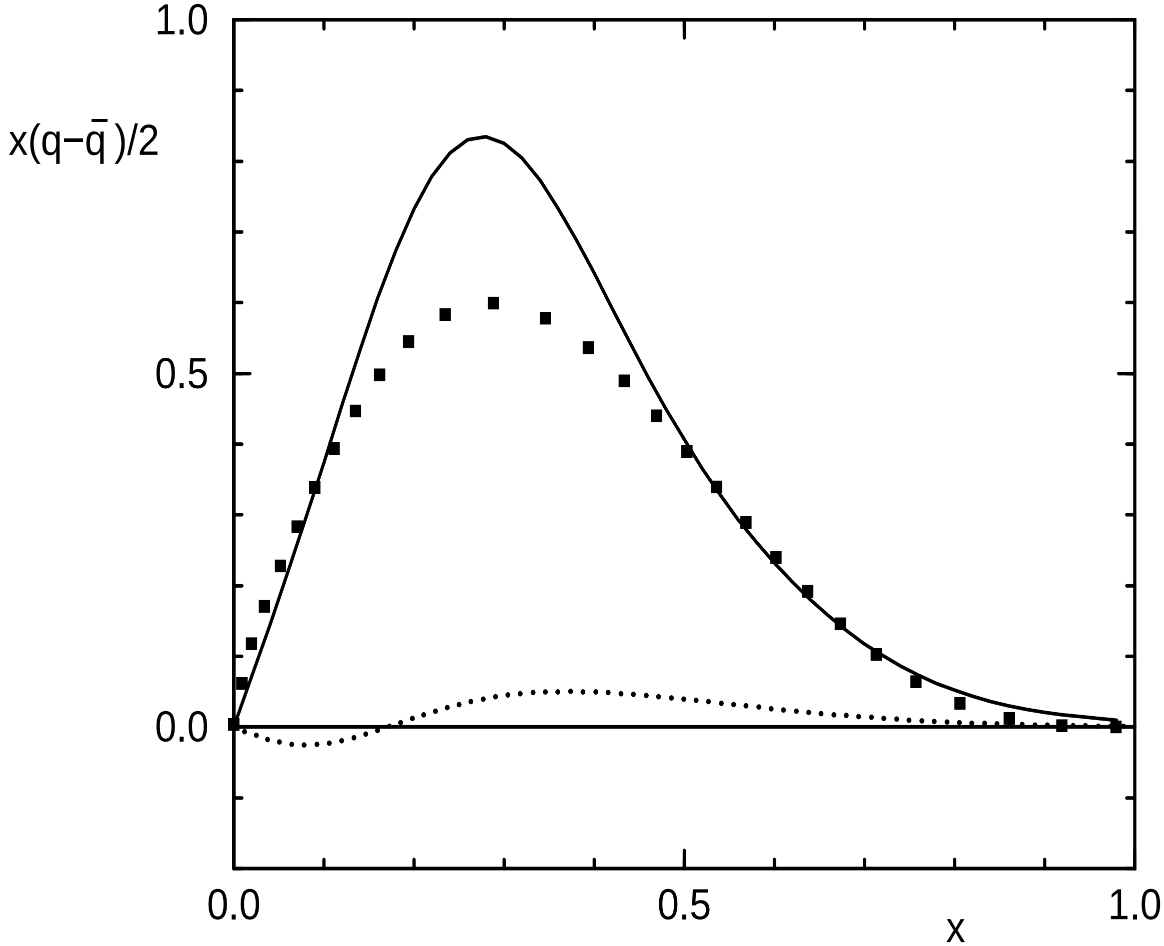


Figure 2: The baryon number distribution, $x[u(x)+d(x)-\bar{u}(x)-\bar{d}(x)]/2$. Solid line: distribution from the unregularized discrete level, eq. (B.5); dotted line: exact Dirac continuum contribution; squares: the parametrization of ref. [15].

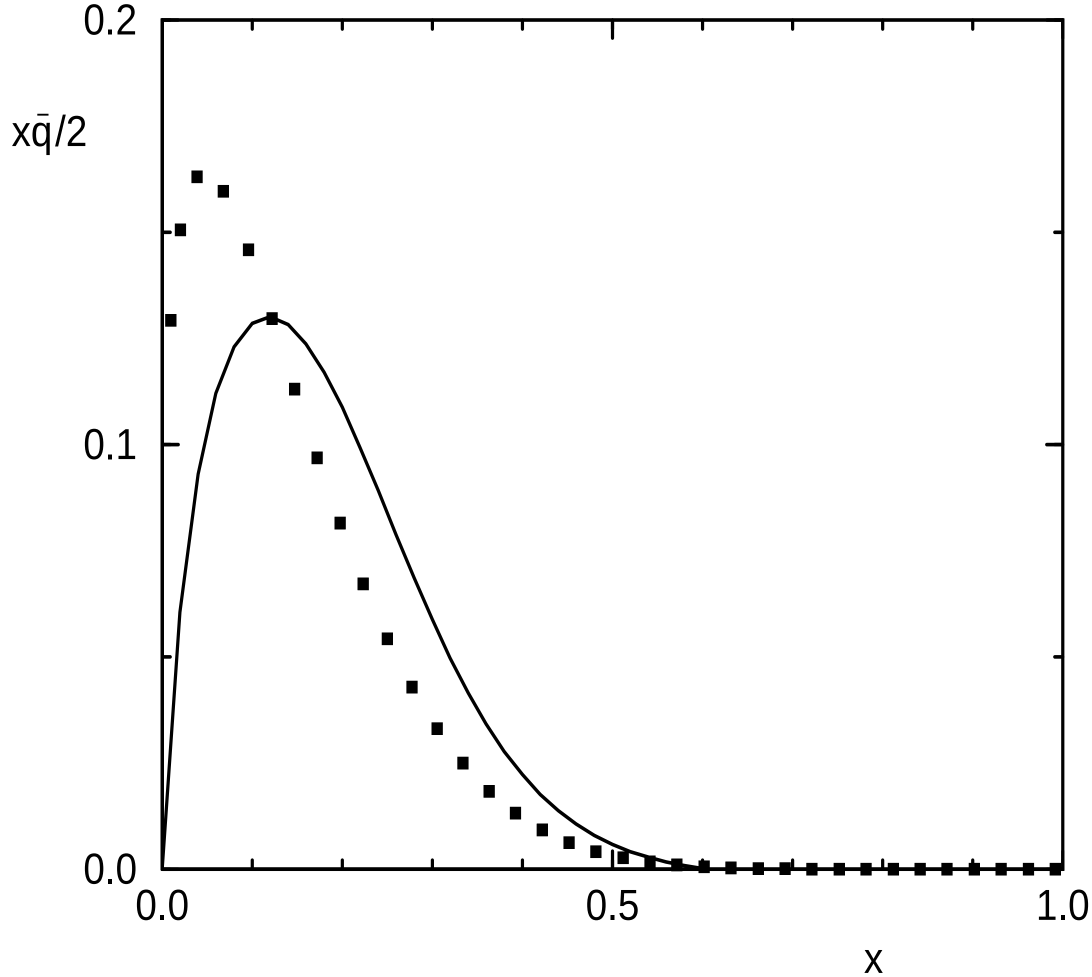


Figure 3: The antiquark distribution, $x[\bar{u}(x) + \bar{d}(x)]/2$. Solid line: theory; squares: the parametrization of ref. [15].

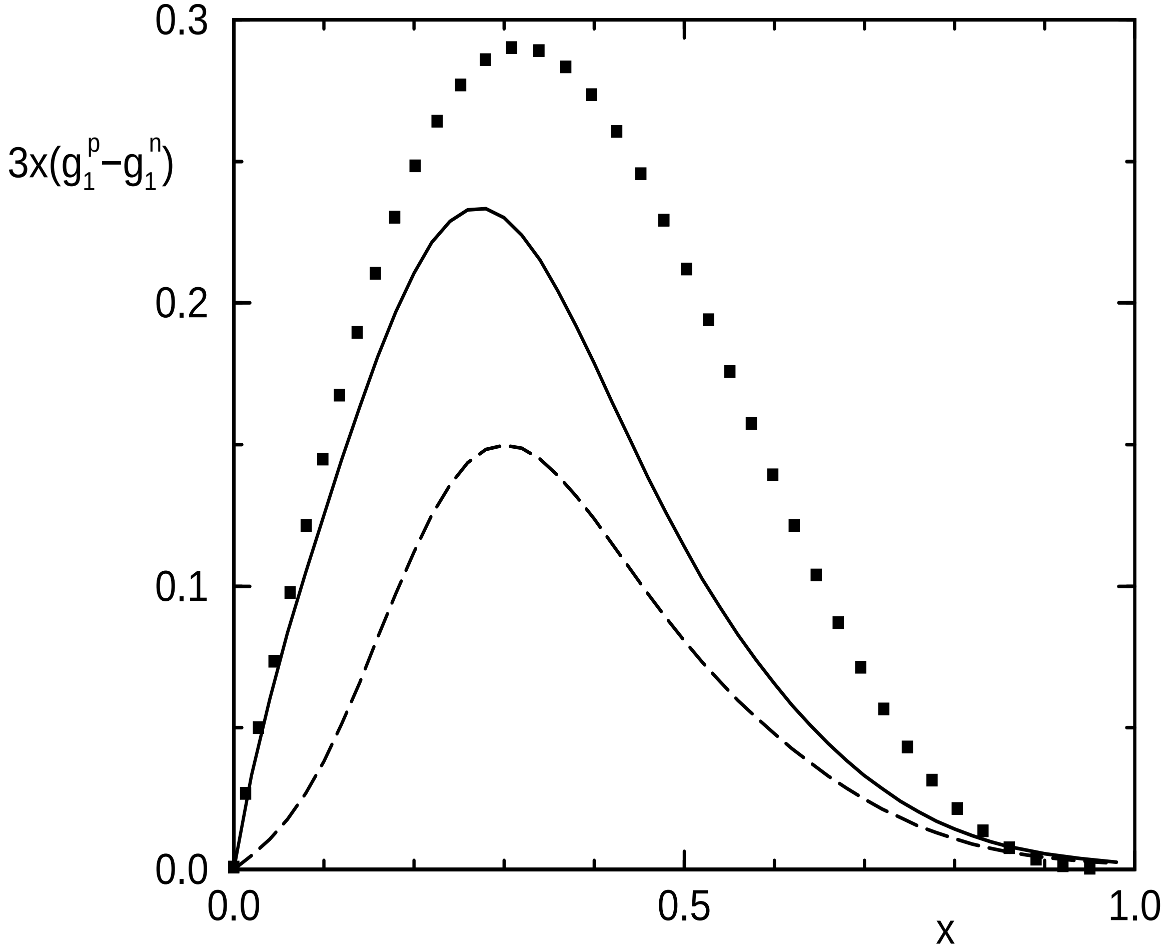


Figure 4: The isovector polarized distribution, $x[\Delta u(x) - \Delta d(x) + \Delta \bar{u}(x) - \Delta \bar{d}(x)]/2$. Dashed line: regularized contribution from the discrete level; solid line: the sum of the contributions from the discrete level and from the continuum according to eq. (7.12); squares: the parametrization of ref. [17].

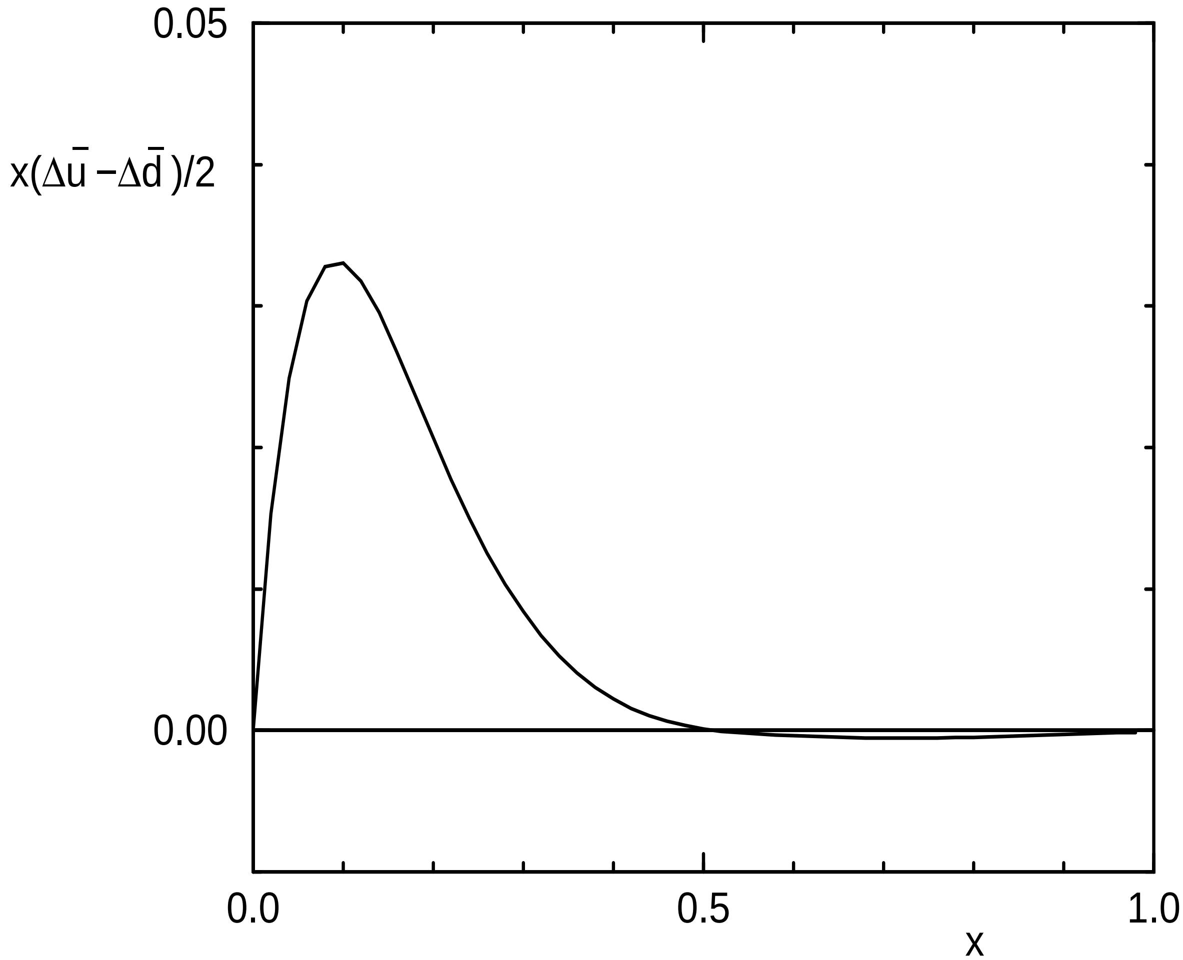


Figure 5: The isovector polarized distribution of antiquarks, $x[\Delta\bar{u}(x) - \Delta\bar{d}(x)]/2$. Ref. [17] assumes this quantity to be zero.